ABSTRACTS

Expander graphs, thin groups and superstrong approximation

Alexander Gamburd ✦ CUNY and IIAS

After briefly discussing classical results on expander graphs we will talk about recent developments pertaining to establishing the expansion property for congruence quotients of thin groups–discrete subgroups of semi-simple groups which are Zariski dense but of infinite index. This expansion property can be viewed as a far-reaching generalization of the strong approximation theorem and has many applications.

The Subspace Theorem of Schmidt and certain of its applications

Umberto Zannier ✦ Scuola Normale Superiore di Pisa

The Subspace Theorem of Schmidt is a far-reaching higher dimensional generalization of Roth's theorem in Diophantine Approximation.
In these two lectures we shall summarise some classical versions and review some applications to equations in integers and other problems. We shall (very briefly) recall classical applications and more recent ones, including for instance one with Corvaja-Rudnick in the context of dynamics of toral automorphisms, and one by Miles to a conjecture of Lind for zeta functions of Z^d-actions.

High-frequency Maass Forms on the modular surface

Peter Sarnak ✦ Princeton University, IAS Princeton and IIAS

We review some of the tools from number theory and dynamics that have been used to study the fine behavior of Hecke-Maass forms in the large frequency (that is semiclassical) limit. In particular we will highlight recent results concerning restrictions of the Maass forms to curves and applications to counting nodal domains.

Diagonalizable actions and number theory

Elon Lindenstrauss ✦ The Hebrew University

Number theoretic applications of homogeneous dynamics typically involve the study of individual orbits. While the qualitative theory of unipotent orbits are fairly well understood, those of diagonalizable actions are less so. A key in both the study of unipotent and diagonalizable actions is the study of invariant measures for the action. For unipotent groups, a complete classification of invariant measures has been given by Ratner, a result that has found numerous spectacular applications. For diagonalizable groups the strongest partial results obtained to date involve either explicitly or implicitly the concept of measure theoretic entropy.
I will survey some of the results in this direction as well as some applications, specifically regarding Linnik-type questions on the distribution of periodic orbits on homogenous spaces.

Toral eigenfunctions and their nodal sets

Jean Bourgain ✦IAS Princeton

In studying spectral aspects of smooth manifolds, the flat torus is perhaps the simplest case because eigenfunctions are completely explicit. There is also a mysterious analogy with the largely unproven phenomena conjectured in the hyperbolic case. Although there is a better analytic grip in the torus case, the most basic problems around the distribution of eigenfunctions and nodal sets turn out to be very hard and the partial progress made relies on diverse areas, including incidence geometry, diophantine analysis, the theory of elliptic curves and of course harmonic analysis. The purpose of the talk will be to give some impression of the various issues one runs into.

Arithmetic and functional transcendence around Schanuel's conjecture

Jonathan Pila ✦ University of Oxford

I will describe the Zilber-Pink conjecture, a far-reaching descendent of the Mordell conjecture which includes the Mordell-Lang conjecture (Faltings's theorem) and the Andre-Oort conjecture as very special cases. I will describe its connections with Schanuel's conjecture, as well as functional versions of Schanuel's conjecture which play a key role in certain approaches to Zilber-Pink problems. This will involve joint work with Jacob Tsimerman and Philipp Habegger.

Congruence subgroups of arithmetic lattices and the limit multiplicity property

Tobias Finis ✦ Freie Universität Berlin

We study the limiting behavior of the discrete spectra of the congruence subgroups of an irreducible arithmetic lattice in a semisimple Lie group G. Assuming that the subgroups in question do not contain any non-trivial central elements, one expects their suitably normalized spectra to converge to the Plancherel measure of G (the limit multiplicity property). We are able to prove this property for the lattices SL(n,O), where O is the ring of integers in a number field, and obtain conditional results in the general case. The focus lies on the case of non-compact quotients, where the spectra have a continuous part. There are two main parts of the proof, which is based on Arthur's trace formula. First, we prove some general results on congruence subgroups of arithmetic lattices and derive bounds on the number of fixed points of non-central elements in the corresponding finite permutation representations. Second, we reduce the control of the continuous spectrum to two properties of intertwining operators, one global and one local, which we can verify for the groups GL(n). This is a joint work with Erez Lapid (Weizmann Institute and The Hebrew University) and partly with Werner Müller (University of Bonn).

Spectral gap estimates for random walks

Péter Varjú ✦ University of Cambridge

Let Γ be a finitely generated subgroup of SL(n, **Z**). Denote by $\Gamma(q)$ the congruence subgroup defined as follows: g is in $\Gamma(q)$ if and only if g is in Γ and g is congruent to Id mod q. Fix a symmetric generating set $S=\{s_1 \ldots, s_k\}$ for Γ, and consider the following averaging operator defined on $L^2(\Gamma/\Gamma(q))$:

$T_q f(g) = 1/|S|(f(s_1 g) + \ldots + f(s_k g))$. We discuss the problem of estimating the largest non-trivial eigenvalue of the operator T_q.

Expansion of random graphs: New proofs, new results

Doron Puder ✦The Hebrew University

We present a new approach to showing that random graphs are nearly optimal expanders. This approach is based on deep results from combinatorial group theory. It applies to both regular and irregular random graphs. Let G be a random d - regular graph on n vertices. It was conjectured by Alon (86') and proved by Friedman (08') in a 100 +/- page-long booklet that the highest non-trivial eigenvalue of G is a.a.s. arbitrarily close to $2\sqrt{d-1}$. We give a new, substantially simpler proof, that nearly recovers Friedman's result. This approach also has the advantage of applying to a more general model of random graphs, concerning also non-regular graphs. Friedman (2003) extended Alon's conjecture to this general case, and we obtain new, nearly optimal results here too.

Binary aspects of the Moebius function and the primes

Jean Bourgain ✦ IAS Princeton

There are several, not clearly related, notions of complexity. In this talk we discuss aspects of symbolic and circuit complexity for the Moebius and Von Mangoldt function. More specifically, problems related to the "Moebius randomness law" and the correlation with the output of certain Boolean circuits. A central issue is the Fourier-Walsh spectrum, closely related to a classical theme, initiated by Sierpinski, of establishing a Prime Number Theorem under certain digital constraints.

Special divisors on hyperelliptic curves

Jacob Tsimerman ✦Harvard University

Let X(1) be the moduli space of elliptic curves over **C**. Points on X(1) corresponding to elliptic curves with complex multiplication are known as Heegner points. The Andre-Oort conjecture - which was proven in this case by Pila (2009) - describes how products of Heegner points are distributed in $X(1)^n$ for the Zariski topology. We will first describe a strengthening of this conjecture by Zhang which describes how Galois orbits of Heegner points are distributed in $X(1)^n$ for the Euclidean topology. We will then explain a natural function field analogue to Zhang's conjecture. In the simplest case, this analogue has an interpretation in terms of counting certain line bundles on hyper-elliptic curves over finite fields, and establishing it amounts to estimating the number of points on intersections of the theta divisor and its translates. Using intersection cohomology methods we are able to reduce the conjecture to an assertion that the total cohomology of such an intersection is exponentially bounded by the genus, and time permitting we explain how to prove this conjecture over the complex numbers. This is a joint work with Vivek Shende.

Adelic action on automorphic periods and special values of L-functions

Andre Reznikov ✦ Bar-Ilan University, Israel

We are interested in invariant functionals defined on automorphic representations via period integrals. We consider the action of an adelic subgroup on such an invariant functional. We show that in certain cases this action gives rise to another period integral, and this corresponds to a known relation of an automorphic period to a special value of an appropriate L-function (that is classical formulas of Hecke-Jacquet-Langlands and of Waldspurger among others). However, even in some of the simplest cases, we find that the relation to L-functions is more puzzling. Namely, the construction leads to a non-standard Euler product which nevertheless could be regularized by an appropriate L-function.
(Joint with J. Bernstein)

Singularities and large values of automorphic forms

Nicolas Templier ✦ Princeton University

We establish lower bounds on the sup-norm of Hecke-Maass forms on GL(n). The argument relies on uniform estimates for Whittaker functions which are of independent interest. For GL(2, Q_p) we establish a new formula that involves ${}_2F_1$ exponential sums. For GL(3,R) we show that the extreme values of Jacquet-Whittaker functions are governed by the Pearcey function. For GL(n;R) we establish a weighted L^2 estimate.
This is a joint work with Farrell Brumley.

Nodal domains and Eigenfunctions of negatively curved surfaces

Junehyuk Jung ✦ Kaist Daejeon

In this talk I'll discuss the nodal set (the zero set) and the nodal domains of eigenfunctions on negatively curved surfaces. By giving a graph structure on the nodal set and using the Euler's inequality for embedded graph, we show that the number of nodal domains is bounded from below by the number of certain singular points of the eigenfunction. The number of such points can be understood by combining recent results on Quantum Ergodic Restriction Theorems and generalized Kuznecov sum formulae. This is a joint work with Steve Zelditch.

Twisted Bhargava's cubes

Wee Teck Gan ✦ National University of Singapore

In his thesis work from 12 years ago, Manjul Bhargava has generalised Gauss' composition law for binary quadratic forms to the higher degree forms. A central example in his theory is the study of a particular prehomogenous vector space: the natural action of SL(2)× SL(2)×SL(2) on $F^2 \otimes F^2 \otimes F^2$, especially the determination of the generic orbits on this space. This prehomogeneous vector space occurs naturally in the split Spin(8). In this talk, I will discuss the orbit problem for a twist of this prehomogeneous vector space, which arises in a quasi-split Spin(8).
This is a joint work with Gordan Savin.

Analytic number theory over finite fields

Emmanuel Kowalski ✦ ETH Zürich

Trace functions of ℓ-adic sheaves have applications in many natural problems of analytic number theory, but their study suggests also a number of new questions with an interesting mixture of analytic and arithmetic aspects. The talk will present some of these, such as counting problems, applications of quasi-orthogonality, and algebraic continuity results for the analogues of certain integral transforms.
Joint work with E. Fouvry and Ph. Michel.

Subconvex bounds for Rankin-Selberg L-function

Paul Nelson ✦ EPFL

For Rankin-Selberg L-functions on GL(2) over a number field, we establish bounds that are uniformly subconvex away from certain explicitly identified "bad" cases. We deduce these from more general estimates for triple product and twisted Asai L-functions.

L^2 restrictions of maass forms

Xiaoqing Li ✦ University of Buffalo

In this talk, we will discuss L^2 restriction norms of Maass forms on higher rank groups. Especially we are interested in lower bounds. We will highlight the distinct properties of higher rank groups.

On the Bateman-Horn conjecture for polynomials over large finite fields

Alexei Entin ✦ Tel Aviv University

The classical Bateman-Horn conjecture predicts the frequency at which the values of several fixed polynomials at an integer are all prime. We prove an analogue of this conjecture for the ring of polynomials over a large finite field.

Affine sieve level

Alex Kontorovich ✦ Yale University and IAS

We will discuss recent progress on the Affine Sieve, highlighting particular instances in which one can obtain levels of distribution beyond those coming from expansion alone.

Recent progress on gaps between primes

Kannan Soundararajan ✦ Stanford University

In the last ten years we have seen astonishing progress on classical problems on prime numbers culminating in the very recent work of Zhang and Maynard. I will explain the ideas behind these works.

On the analytic theory of Frobenius trace functions

Philippe Michel ✦ EPFL

Frobenius trace functions are arithmetic functions periodic of period p a prime number. These are obtained from constructible ι-sheaves, mixed of weight $\leqslant 0$ on the affine line over the finite field F_p. In this lecture we discuss the correlations between such functions and other arithmetic functions like the characteristic function of short intervals, of the primes or the Fourier Coefficient of modular forms and exhibit necessary conditions on the underlying lisse geometric sheaf to guarantee the absence of correlation. We will also describe some applications of these techniques to study of the distribution of primes in large arithmetic progression. This is a collection of joint works with E. Kowalski, E. Fouvry, Paul Nelson as well as DHJ Polymath8.

Torsion homology growth and cycle complexity of arithmetic manifolds

Nicolas Bergeron ✦ UPMC-IMJ

Let M be an arithmetic hyperbolic 3-manifold, such as a Bianchi manifold. in this talk I will state a conjecture which predicts that there is a basis for the second homology of M, where each basis element is represented by a surface of "low" genus. I will then give some evidence for this conjecture and explain its relationship with the study of torsion homology growth. This is a joint work with Mehmet Haluk Şengün and Akshay Venkatesh.

Zeros of the derivative of the Riemann zeta-function

Stephen Lester ✦ Tel Aviv University

The distribution of the zeros of the derivative of the Riemann zeta function is closely related to the distribution of the zeros of ζ (s). For instance, a result of Speiser states that the Riemann hypothesis is equivalent to $\zeta'(s)$ not vanishing in the strip $0< \mathrm{Re}(s)\frac{1}{2}$. In this talk we will elaborate further upon the connections between zeros of $\zeta'(s)$ and zeros of $\zeta(s)$. We will also discuss the known properties of zeros of $\zeta'(s)$ and mention some recent progress in describing their distribution.

On the nodal sets of random band limited functions

Igor Wigman ✦ King's College London

We consider some nodal aspects of random Gaussian band-limited functions on a Riemannian manifold. These include some important examples, (such as Random Spherical Harmonics, "Complex Fubini-Study" and "Real Fubini-Study"). Using techniques based on the powerful methods recently developed by Nazarov and Sodin we establish the semi-locality and the existence of a universal law for a number of properties, such as the homeomorphy classes of nodal components, and their connectivity graphs in the high-energy limit.
This is a joint work with Peter Sarnak.

A refined non-vanishing theorem for the central L-values

Wenzhi Luo ✦ The Ohio State University

The non-vanishing as well as the size of the central L-values for newforms with large weights has deep arithmetic implications. For instance it has an intrinsic connection with the Landau-Siegel zeros in the analytic theory of Dirichlet series as shown in the well-known work of Iwaniec and Sarnak.
These central L-values are usually studied via averaging over the weight via explicit asymptotic evaluation of the Neumann series, thus bypassing the intricate issue of phase transition of Bessel function involved in the trace formula. In this talk, we will explain how to derive a positive proportion non-vanishing result for the central L-values with individual large weight. As applications, we prove some quantitative results on the simultaneous non-vanishing of the central L-values and their twists by quadratic characters, for pairs of newforms with the same weight.

Height gap versus spectral gap

Emmanuel Breuillard ✦ Université Paris-Sud 11

I will describe several instances in which height lower bounds are intimately related to spectral gaps for group representations.

Low-lying zeros of elliptic curve L-functions

Anders Södergren ✦ University of Copenhagen

In this talk we will study the distribution of low-lying zeros of the family of L-functions attached to quadratic twists of a given elliptic curve. We will describe how a technique of Katz and Sarnak can be used to give very precise information about the corresponding 1-level density. In particular, for test functions whose Fourier transforms have sufficiently restricted support, we are able to compute the 1-level density up to an error term that is significantly sharper than the square root cancellation predicted by the L-functions Ratios Conjecture.
This is a joint work with D. Fiorilli and J. Parks.

From Ramanujan graphs to Ramanujan complexes

Alex Lubotzky ✦ The Hebrew University

Ramanujan graphs are optimal expanders (from a spectral point of view). Explicit constructions of such graphs were given in the 1980s as quotients of the Bruhat-Tits tree associated with GL(2) over a local field F, by suitable congruence subgroups. The spectral bounds were proved using works of Hecke, Deligne and Drinfeld on the "Ramanujan conjecture" in the theory of automorphic forms. The work of Lafforgue, extending Drinfeld from GL(2) to GL(n), opened the door for the construction of Ramanujan complexes as quotients of the Bruhat-Tits buildings. This gives finite simplical complexes which on one hand are "random like" and at the same time have strong symmetries. Recently various applications have been found in combinatorics, coding theory and in relation to Gromov's overlapping properties. We will describe these developments and give some details on recent applications. The work of a number of authors will be surveyed. Our works in these directions are in collaboration with various subsets of: S. Evra, K. Golubev, T. Kaufman, D. Kazhdan, R. Meshulam, S. Mozes, B. Samuels and U. Vishne.