The 20th Midrasha Mathematicae

60 Faces to Groups

Abstracts

Survey of representation growth
Nir Avni - Northwestern University

Higher-rank lattices have finitely many irreducible representations of any given dimension. 15 years ago Alex Lubotzky posed the problem of counting the number of such representations. I will talk about the partial answers we know at present.

Finite permutation groups and the Graph Isomorphism problem
László Babai - University of Chicago

The Graph Isomorphism problem is the algorithmic problem to decide whether or not two given finite graphs are isomorphic. Recent work by the speaker has brought the worst-case complexity of this problem down from $\exp(\sqrt{n \log n})$ (Luks, 1983) to quasipolynomial ($\exp((\log n)^c)$), where n is the number of vertices. Group theory is behind all ingredients of this work.

- The problem we actually address is the equivalence of strings under a permutation group action (Luks, 1980);
- The core mathematical result underlying the algorithm is a lemma on actions of permutation groups ("Unaffected stablizers lemma");
- The central algorithm investigates a projection of an unkown automorphism group ("Local certificates algorithm");
- The main combinatorial partitioning algorithms ("Design Lemma" and "Split-or-Johnson") operate on highly regular structures called "*coherent configurations.*" This concept is a combinatorial abstraction of the orbital structure of permutation groups, going back to Schur (1933).

We shall explain these ingredients and discuss the group theoretic aspects in some detail. Time permitting, we shall also comment on the related "Group Isomorphism problem" that highlights a major barrier to further progress.

Locally testable groups
Oren Becker - The Hebrew University of Jerusalem

Arzhantseva and Paunescu [AP2015] showed that if two permutations X and Y in Sym(n) nearly commute (i.e. XY is close to YX), then the pair (X,Y) is close to a pair of permutations that really commute. We regard this result as a property of the equation ab=ba and say that this equation is "locally testable" (aka stable). The general question is: which equations (or sets of equations) are locally testable. One sees that the answer depends only on the abstract group defined by the equations (as relations on the generators) rather than on the equations themselves. This leads to the notion of "locally testable groups" (aka "stable groups"). So, the main result of [AP] is that that abelian groups are locally testable, while Glebsky and Rivera [GP2009] showed that the same is true for every finite group. In addition, [GR] show that a locally testable sofic group must be residually finite. In a joint work with Alex Lubotzky (work in progress), we develop an ensemble of tools to check whether or not a given group is locally testable. As an application, we can answer two questions of [AP] by showing that the Baumslag-Solitar group BS(1,2) is locally testable and that there exists a finitely presented solvable (hence amenable) residually finite group which is not locally testable. In addition, we prove that sofic groups with property (T) are not locally testable, and that the discrete Heisenberg group is locally testable.

Strongly dense free subgroups
Emmanuel Breuillard - University of Muenster

A subgroup Gamma of a semisimple algebraic group G is called strongly dense if every subgroup of Gamma is either cyclic or Zariski-dense. I will describe a method for building strongly dense free subgroups inside a given Zariski-dense subgroup Gamma of G, thus providing a refinement of the Tits alternative. The method works for a large class of G's and Gamma's. I will also discuss connections with word maps, expander graphs, and the Banach-Tarski paradox. This is joint work with Bob Guralnick and Michael Larsen.

Generation and Presentations: Kac-Moody groups' perspective
Inna Capdebocsq - University of Warwick

In this talk we discuss results about generation and presentations of Kac-Moody groups over finite fields, and their consequences for some Chevalley groups. This talk is partly based on a joint work with A. Lubotzky and B. Remy.

Zeta functions of Groups
Marcus du Sautoy - University of Oxford

Ever since Riemann's seminal paper on the primes, the zeta function has proved a powerful weapon in the mathematician's arsenal. In recent years, group theorists have discovered that non-commutative analogues of classical zeta functions in number theory provide an interesting new perspective on the theory of infinite groups. These zeta functions encode in a Dirichlet series arithmetic information about the lattice of subgroups of an infinite group. This lecture will survey some of the new insights that these zeta functions have given group theorists.

The Congruence Subgroup Problem for Free Metabelian groups
David El-Chai Ben-Ezra - The Hebrew University of Jerusalem

The classical congruence subgroup problem, in its modern setting, asks whether the congruence map $\widehat{GL_n}(Z) \to GL_n(\hat{Z})$ is injective, and if not, what is its kernel C_n (here, $\hat{X}$ denotes the profinite completion of X). It is known that there is a dichotomy between $n = 2$ where C_n is huge and isomorphic to the free profinite group on countable many generators, and between $n \geq 3$ where C_n is trivial.

Relating to $GL_n(Z)$ as the automorphism group of $\Gamma = Z^n$, the free abelian group on n generators, one can generalize the congruence subgroup problem to the automorphism group of arbitrary finitely generated group Γ, and ask, what is the kernel of the map $\widehat{Aut}(\Gamma) \to Aut(\hat{\Gamma})$. Considering this generalization, very few results are known when Γ is not abelian. For example, we do not even know to describe the congruence kernel for $\Gamma = F_n$, the free group on n generators, when $n \geq 3$.

On the talk, I will present some recent results, showing that when Γ is the free metabelian group on n generators, we have a dichotomy between $n = 2, 3$ and $n \geq 4$.

Topological Expanders
Shai Evra - The Hebrew University of Jerusalem

A classical result of Boros-Furedi (for $d = 2$) and Barany (for $d \geq 2$) from the 80's, asserts that given any n points in R^d, there exists a point in R^d which is covered by a constant fraction (independent of n) of all the geometric (=affine) d-simplices defined by the n points. In 2010, Gromov strengthen this result, by allowing to take topological d-simplices as well, i.e. drawing continuous lines between the n points, rather then straight lines and similarly continuous simplices rather than affine. He changes the perspective of these questions, by considering the above results as a result about geometric/topological expansion properties of the complete d-dimensional simplicial complex on n vertices. He asked whether there exits bounded degree simplicial complexes with the above geometric/topological properties, i.e.

"bounded degree geometric/topological expanders". The geometric problem was answered in 2013 by Fox-Gromov-Lafforgue-Naor-Pach who showed that the Ramanujan complexes constructed by Lubotzky-Samuels-Vishne give rise to such geometric expanders, but left open the more difficult topological question. This question was answer affirmatively for dimension $d = 2$, by Kaufman, Kazhdan and Lubotzky. By extending the method of proof of Kaufman, Kazhdan and Lubotzky: we show that the $(d-1)$-skeletons of the d-dimensional Ramanujan complexes give bounded degree topological expanders. This is joint work with Tali Kaufman.

Ramsey Theory for Non-amenable Groups
Hillel Furstenberg - The Hebrew University of Jerusalem

Ramsey theory for groups is the study of notions of "largeness" for subsets of a group, and of structures that necessarily occur in large sets. Szemeredi's theorem for the integers is an example of this. Such results are generally easily extended to amenable groups for which the ergodic approach is available. For non-amenable groups these methods are not applicable directly but variants can be found. An example is a condition on subsets of the free group on any number of generators that guarantees existence of geometric progressions of arbitrary length.

Models of random group via the dynamical constructions on the space of groups
Rostislav Grigorchuk - Texas A&M University

I will remind the construction of the space of finitely generated group and discuss the action of a group $\mathrm{Aut}(F_\infty)$ of automorphisms of a free group on it. Then I will suggest a dynamical approach to the notion of a random group and provide some examples. Finally, I will relate this topic to the topic of invariant random subgroups and will state few results in this direction. The talk will be based on the old results of the speaker, and recent results obtained in collaboration with M. Benli, L. Bowen, R. Kravchenko and Y. Vorobets.

Presentations of finite groups and cohomology
Robert Guralnick - University of Southern California

Let G be a finite group. We will discuss the problem of determining presentations for G with a small number of relations and with short relations. These problems are also related to giving bounds for low degree cohomology groups. We will mostly discuss the case where G is close to a simple group (which is the critical case). We will discuss some work with Kantor, Kassabov and Lubotzky as well as more recent work.

From character varieties to quantum representation of mapping class groups
Martin Kassabov - Cornell University

I will describe a new way at looking at representation and character varieties as tensor product over suitable category. One of the advantages of this view point is that it provides an easy way to derive this functor and define representation homology. An other advantage is that by deforming the category, one can define deformation of the character varieties of fundamental groups of 3 manifolds. The can be extended to TQFT (which considers with a skein theoretic one), and provides an easy way to define quantum representation of mapping class. I hope that this construction of quantum representation will lead to simplified proof of some of their properties, like asymptotic faithfulness. (joint work with S. Patotski)

Recognizing groups in arithmetic problems
Emmanuel Kowalski - ETH Zürich

The solution of some important arithmetic questions relies on the concrete identification of groups that appear as Galois groups (or monodromy groups). We will survey some of the techniques that exist to solve this problem, many developed by N. Katz over many years, emphasizing the parallel between problems involving finite groups and algebraic groups. A recent application concerns a geometric analogue of the Schinzel Hypothesis in prime number theory; we will sketch some results and also discuss interesting problems that arise out of this analogy.

Positively finitely generated groups and their associated ζ-functions
Avinoam Mann - The Hebrew University of Jerusalem

A group is *positively finitely generated* **PFG** if it can be finitely generated with positive probability. We will first make this notion precise, this is done in the category of profinite groups. It turns out that this is a very large class of groups, including, e.g., prosoluble groups and arithmetic groups with the **CSP**, and many more, but does not include free (not cyclic) profinite groups. The **PFG** groups can be characterized in terms of their subgroup growth (Mann – Shalev) or in terms of multiplicities of chief factors (Jaikin-Zapirain – Pyber). Conjecturally, the generation probabilities of a group G can be interpolated to an analytic function, which gives rise to the *probabilistic ζ-function* of G. The existence of this function was proved in many cases, e.g. for the aforementioned classes. When this function exists, it has an Euler factorization, indexed by the finite simple groups.

Algorithms for finitely presented groups, and probabilistic group theory
Martin Liebeck - Imperial College London

According to a result of Bridson and Wilton, there is no algorithm that, given a finite presentation, determines whether or not the group presented has a nontrivial finite quotient. Consequently there no algorithm that determines the finite simple images of a finitely presented group. On the other hand, Plesken, Fabianska and Jambor have given an algorithm that determines the images of a finitely presented group that are isomorphic to $\mathrm{PSL}_2(q)$ or $\mathrm{PGL}_2(q)$ for some q, and Jambor has done the same for 3-dimensional classical groups. The question arises – for which collections of finite simple groups is there an algorithm that determines the members of the collection that are images of an arbitrary finitely presented group G? I shall present a partial solution to this problem. The methods involve probabilistic group theory, model theory and representation theory.

High Dimensional Expansion
Roy Meshulam - Technion

Expander graphs have been a focus of intensive research in the last four decades, with numerous applications throughout mathematics and theoretical computer science. In view of the ubiquity of expander graphs, there is recent growing interest in high dimensional versions of expansion, such as spectral gaps of higher Laplacians and coboundary expansion. Following his well-known seminal contributions to graphical expansion, in recent years Alex and his collaborators have obtained numerous fundamental results in the area of high dimensional expansion. In this talk I'll survey some aspects of the emerging theory and its connections with topological combinatorics.

On the growth of torsion in homology of finite index subgroups
Nikolay Nikolov - University of Oxford

There is a lot of interest regarding the growth of invariants of chains of finite index subgroups, e.g. the growth of Betti numbers, rank, deficiency and so on. In this talk I will consider the growth of another invariant: the size of the torsion subgroup in homology. I will focus on two main classes of groups where there has been recent progress: amenable groups (joint work with Kar and Kropholler) and right angled groups (joint work with Abert and Gelander). The main tools are from combinatorial group theory, ergodic theory and the notion of combinatorial cost

Golden Gates and High Dimensional Expanders
Ori Parzanchevski - The Hebrew University of Jerusalem

Some thirty years ago, Lubotzky, Phillips and Sarnak initiated the theory of Ramanujan graphs, bringing deep insights from number theory into combinatorics and computer science. Recently, their work found a new application in the construction of "Golden Gates" - optimal gates for quantum computations with a single qubit. In the last decade, Alex's attention has shifted to higher dimensions, studying the combinatorics and topology of Ramanujan complexes. We will explain what these new structures are, and how they shed light on the construction of golden gates for more than one qubit.

How to avoid the Classification Theorem of Finite Simple Groups in Asymptotic Group Theory
László Pyber - Alfréd Rényi Institute of Mathematics, H.A.S.

The Classification of Finite Simple Groups (CFSG) is a monumental achievement and a seemingly indispensable tool in modern finite group theory. By now there are a few results which can be used to bypass this tool in a number of cases, most notably a theorem of Larsen and Pink which describes the structure of finite linear groups of bounded dimension over finite fields.

In a few cases more ad hoc arguments can be used to delete the use of CFSG from the proofs of significant results. The talk will among others discuss a recent example due to the speaker: how to obtain a CFSG-free version of Babai's quasipolynomial Graph Isomorphism algorithm by proving a Weird Lemma about permutation groups.

Eigenvalue Rigidity and Hearing the Shape of a Locally Symmetric Space
Andrei Rapinchuk - University of Virginia

I will discuss the notion of weakly commensurable Zariski-dense subgroups which was introduced and analyzed in a joint work with G. Prasad. It turns out that absolutely almost simple algebraic groups containing such subgroups must share some important characteristics - we call this phenomenon "eigenvalue rigidity" as weak commensurability is based on matching the eigenvalues of elements. I will then report on how our results on weakly commensurable arithmetic groups were used to resolve the famous question "Can one hear the shape of a drum?" for many locally symmetric spaces. Time permitting, I will also mention some aspects of the ongoing project (joint with V. Chernousov and I. Rapinchuk) that involves not necessarily arithmetic groups and leads to problems in the theory of algebraic groups of independent interest.

Profinite rigidity in low dimensions
Alan Reid - University of Texas at Austin

A finitely generated residually finite group G is called *profinitely rigid* if whenever a finitely generated residually finite group H satisfies $\widehat{H} \cong \widehat{G}$, then $H \cong G$ (where $\widehat{G}$ denotes the profinite completion). Although by now there are many examples of groups that are not profinitely rigid, there seems to be a growing sense that when G is a free group, surface group or the fundamental group of a finite volume hyperbolic 3-manifold, things are different and these will be profinitely rigid. In this we will describe recent progress on this topic.

Linear groups and simple groups
Aner Shalev - The Hebrew University of Jerusalem

The talk will focus on recent results in the study of objects to which Alex contributed very significantly over the years. We will present a probabilistic Tits alternative for linear groups, a rapid growth result for finite simple groups, and a proof of recent conjectures of Gowers and Viola on mixing and complexity in finite simple groups. If time permits we will also discuss applications to simple algebraic groups and a question of Hrushovski on the irreducibility of certain subsets.

Navigating $\mathrm{PU}(2)$, Golden Gates and Strong Approximation
Peter Sarnak - IAS and Princeton University

We discuss recent developments concerning "Golden Gates" which are number theoretic generators of PU(2) ,their application to the construction of optimally efficient universal quantum gates ,and some closely connected questions of complexity in strong approximation. **The lecture will take place at the mathematica building, lecture hall 2**.

Strongly complete profinite groups
Dan Segal - University of Oxford

A profinite group is *strongly complete* if all its subgroups of finite index are open. It is known that every finitely generated profinite group has this property; I want to explore what, if anything, can be said about these groups without assuming finite generation, and to what extent they can be characterized algebraically.

Finding infinity inside Outer space
Karen Vogtmann - University of Warwick and Cornell

Motivated by work of Borel and Serre on arithmetic groups, Bestvina and Feighn defined a bordification of Outer space; this is an enlargement of Outer space which is highly-connected at infinity and on which the action of $\mathrm{Out}(F_n)$ extends, with compact quotient. They conclude that $\mathrm{Out}(F_n)$ satisfies a type of duality between homology and cohomology. We show that Bestvina and Feighn's bordification can be realized as a deformation retract of Outer space instead of an extension, answering some questions left open by Bestvina and Feighn and considerably simplifying their proof that the bordification is highly connected at infinity.

Infinite infinitesimal groups and their representations
Efim Zelmanov - University of California, San Diego

We will discuss infinite Lie algebras and superalgebras and their representations.
The lecture will take place at the mathematica building, lecture hall 2.