Outline

1. Two kinds of display ads
   - Brand versus performance advertising
2. Contractual set-asides: costs & benefits
   - Avoid adverse selection, lose matching value
3. Model and Axiomatic result
   - Deriving the MSB auction
4. Benchmarking tests
   - Compare MSB to set-asides and to OPT
Two Kinds of Display Ads

Display Ads: Brand Advertisers
Contrasting Advertisements

**Performance Display Ads**
- Goal: immediate action
  - Click, fill form, or buy
  - Targeting using cookies
  - Sold one by one
  - Price set by auction
- Sample Advertisers
  - Amazon (re-targeting)
  - Quicken mortgage (refinance)
  - Hertz (car rental)

**Brand Display Ads**
- Goal: awareness or image
  - Reach and repetition
  - Demographic targeting
  - Bought en masse
  - Price set by contract
- Sample Advertisers
  - Disney (movie openings)
  - Shopping Center (general)
  - Ford (weekend auto sale)
Risk of Adverse Selection

“Half the money I spend on advertising is wasted; the trouble is I don’t know which half.” – John Wannamaker (advertising pioneer)

Performance Ads
- Immediate feedback about individual ad performance
- May be based on private user cookies

Brand Ads
- Difficult to detect which ads and websites are performing well.
- Not based on cookies ⇒ an information disadvantage

Contractual Set-Asides: Costs & Benefits

Avoid adverse selection but lose matching value
How Costly Are Set Asides?

80-20 Example

- Assume contracts require that 80% of certain impressions be assigned to brand advertisers, leaving 20% for performance advertisers
- **Notation:** $X = \text{Auction value if 100\% of impressions were sold to performance advertisers}$
- **Assumption:** 80\% of the value of performance advertising is associated with the 20\% most valuable impressions
  - The 80-20 rule is a common marketing rule of thumb, which we assume applies in this example.

Arithmetic of Set Asides

- Random set-aside (RSA): 20\% of impressions are “randomly” chosen for auction: second-price auctions for the rest.
  - Performance advertisers get impressions they value at $0.2X$.
- Alternative (SPR): second-price auction with reserve set so that performance advertisers win most valuable 20\% of impressions.
  - Performance advertisers get impressions they value at $0.8X$.
- Compared to RSA, auctions enable four times more value from performance ads.
- Questions
  - Is the example in some way typical?
  - How would auction gains be shared between publisher and advertisers?
Set-Asides are Pessimal

- More generally, suppose that...
  - there is one brand advertiser and \( N \) performance advertisers.
  - match values are IID draws from a distribution \( F \) with density \( f \).
  - Notation: Myerson’s “virtual value” is \( g(x) = x - (1 - F(x)) / f(x) \).
- Fix the proportion \( \lambda \) of impressions to be awarded to the brand advertiser.
- **Theorem.** Among all symmetric strategy-proof auctions for which the performance advertisers win with probability \( 1 - \lambda \), the RSA mechanism...
  - Minimizes expected total value of performance impressions
  - Minimizes expected seller revenue if virtual value \( g(x) \) is non-decreasing
  - Minimizes expected bidder surplus if \((1 - F(x)) / f(x)\) is non-decreasing

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Proof Sketch

- Three steps
  1. Standard characterizations
     - Seller’s expected revenue = \( E[p(x_{(1)})g(x_{(1)})] \), where \( p(x) \) is the probability that this highest performance bid wins.
     - Bidder’s expected profit = \( E[p(x_{(1)})(1 - F(x_{(1)}) / f(x_{(1)})]. \)
  2. Strategy-proof mechanism \( \Rightarrow p(x) \) is non-decreasing.
  3. **Majorization inequalities:**
     - \( E[p(x_{(1)})x_{(1)}] \geq E[p(x_{(1)})] E[x_{(1)}] = (1 - \lambda) E[x_{(1)}] \)
     - \( E[p(x_{(1)})g(x_{(1)})] \geq E[p(x_{(1)})] E[g(x_{(1)})] = (1 - \lambda) E[g(x_{(1)})] \)
     - \( E[p(x_{(1)})(1 - F(x_{(1)}) / f(x_{(1)})] \geq \ldots = (1 - \lambda) E[(1 - F(x_{(1)}) / f(x_{(1)})] \)
So, why use set-asides?

Key Hypothesis: Adverse selection is a serious enough problem to explain the use of set asides in preference to auctions.

Informal Discussion
- Set-asides are familiar to brand advertisers from other media.
- Without set-asides, brand advertisers would have to bid into auctions based just on public information about impressions.
- Performance advertisers can use more detailed information, including cookies, to select impressions.
- The losses due to adverse selection are large enough to justify resistance to auctions and much higher prices for set-aside impressions.

What Can We Do?

Is it possible to capture most matching value and also avoid adverse selection?

Idea: impressions with high consumer quality may be distinguishable from those with high idiosyncratic match quality.
Private values assumption: Each performance bidder $n$ knows its own value $X_n$ for the impression.

Definition: A display auction is “weakly efficient” if for every bid profile, the impression is awarded either to the brand advertiser or to the highest performance advertiser.

Lemma. In the private values model, a weakly efficient, deterministic mechanism is strategy-proof if and only if it is a threshold auction.

A “threshold auction” sets a price for each bidder as a function of the others’ bids.
Value model for one impression

- **Notation**
  - $B$ is the brand advertiser’s value.
  - $X_n$ is the value for performance bidder $n$.
  - $C = \frac{E[B | X_1, \ldots, X_N]}{E[B]}$ is the “consumer quality”
  - $M = \frac{X}{C}$ is the “match quality vector”

Auction Allocation

- **Notation.** Given a bid profile $x$ from performance bidders,
  - $z_n(x)$ is the probability that a mechanism awards the impression to performance advertiser $n$.
  - $z_0(x)$ is the probability that a mechanism awards the impression to the brand advertiser.
Adverse-Selection Free Auctions

- **Definition.** A mechanism is adverse-selection free if for every distribution of \((M, C)\) such that \(M\) and \(C\) are independent, \(z_0(X)\) is independent of \(C\).

- **Theorem.** A threshold auction is adverse-selection free if and only if the pricing function \(p(\cdot)\) is homogeneous of degree one: \(p(cm) \equiv cp(m)\).

Proof Sketch

- **Sufficiency**
  - If \(p\) is homogeneous of degree one, then the event that the brand advertiser wins has this characterization:
    \[ \{x_{(1)} \leq p(x_{(-1)})\} = \{cm_{(1)} \leq p(cm_{(-1)})\} = \{m_{(1)} \leq \alpha p(m_{(-1)})\} \]
  - This event depends only on \(m\), which is independent of \(c\)

- **Necessity:** routine.
Definitions.

A mechanism is false-name proof if
- no winning bidder can reduce its price by placing additional bids, and
- no seller can increase its price by placing additional very low bids.

A threshold auction is a modified second-bid (MSB) auction if there exists \( \alpha > 1 \) such that \( p(x_n) = \alpha \max(x_n) \).

Theorem. A threshold auction is deterministic, adverse-selection free and false-name proof if and only if it is an MSB auction.

Proof Sketch

The mechanism is false-name proof because it depends only on the two highest bids (and \( N \geq 2 \)), and any mechanism that depends on more bids cannot be false name proof.

The mechanism is adverse selection free if and only if \( p \) is homogenous of degree 1, and this is the only such function.
Benchmarks Tests

Compare MSB to set-asides and to OPT

Power Law Distribution

- **Notation:**
  - $M_{(k)} = k^{th}$ order statistic
  - $r = M_{(1)}/M_{(2)}$.
- For benchmark computations, assume that the variables $M_n$ are drawn IID from the power law distribution with parameter $\alpha$: $\Pr\{M_n > \mu\} = \mu^{-\alpha}$.
- Four implications of the power law distribution.
  1. $\ln(M_n)$ has an exponential distribution with mean $1/\alpha$.
  2. $r$ and $M_{(2)}$ are statistically independent.
  3. $r$ has the power law distribution.
  4. $E[r \mid r > \alpha] = \alpha E[r]$. 
Comparison to Set-Asides

Fat-Tail Assumption: The match values $M_1, \ldots, M_N$ are IID draws from a distribution $F$ with density $f$ and such that $h(x) = xf(x)/(1-F(x))$ is non-increasing.

Remark: For the power law $1-F(x) = x^{-\alpha}$, $f(x) = \alpha x^{-\alpha-1}$ so $h(x) = \alpha$ is a constant.

Theorem. For the MSB with parameter $\alpha > 1$ and all distributions satisfying the fat-tail assumption, bidder profits and seller revenues are at least $\alpha$ times higher than in the random set aside mechanism.

Power Law Case

Using the power law, computations are easy.

- Expected value from performance impressions:
  $EVP = E[C_{M(1)} 1_{(r>\alpha)}] = E[M_{(2)} r 1_{(r>\alpha)}] = E[M_{(2)}] E[r | r > \alpha] \Pr(r > \alpha)$
  $= E[M_{(2)}] \alpha E[r](1-\lambda) = \alpha(1-\lambda) E[M_{(1)}]$

- Expected revenue from performance impressions:
  $ERP = E[\alpha M_{(2)} 1_{(r>\alpha)}] = \alpha E[M_{(2)}] \Pr(r > \alpha) = \alpha(1-\lambda) E[M_{(2)}]$

- Expected bidder profit from performance impression:
  $EPP = EVP - ERP = \alpha(1-\lambda) E[M_{(1)} - M_{(2)}]$
Possible Magnitudes

- Consider again the 80-20 rule.
  - \( E[r1_{r \text{ in top quintile}}] = 0.8x \text{E}[r] \).
  - It corresponds to a power law distribution with \( \alpha = 1.16 \).

- Sample calculations using power law \( \alpha = 1.16 \).
  - Case 1: 80% of ads reserved for brand: \( \lambda = 0.8 \).
    - Infer that performance revenues and profits must increase by 300%, so \( \alpha = 4.0 \).
  - Case 2: 50% of ads reserved for brand: \( \lambda = 0.5 \).
    - Then, \( \alpha = 1.8 \): performance revenues and profits increase by 80%.

Comparison to SPR

- Consider a second-price auction (SPR) in which the reserve price is set so that the auction delivers the same fraction of impressions \( \lambda \) to brand advertisers as does the MSB with parameter \( \alpha \).

- Theorem. If C=1 almost surely, then for all power law distributions, all fixed numbers of bidders \( N \) and all \( \alpha \), the ratio of seller revenues and performance bidder profits under MSB to those in the corresponding second price auction with reserve is no less than 0.886.
End