Can we Make School Choice more Efficient?

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School Choice

Thousands of students in Boston and New York participate in the public school choice system

- Students (strategically) submit a (strict) preference over schools
- Schools (non-strategically) have (weak) priorities over students
- A central mechanism produces the best matching for students that respects priorities

The current mechanism is Pareto-inefficient, but any improving mechanism (allowing trade) will not be strategy-proof.

Literature

DA-STB is inefficient, SOSM is not Strategy-proof: Erdil, Ergin AER 08; Abdulkadiroğlu, Pathak, Roth AER 09

Current Design: Abdulkadiroğlu, Pathak, Roth AER PP 2005 Abdulkadiroğlu, Pathak, Roth, Sönmez AER PP 05

Incentive properties of deferred acceptance: Kojima, Pathak AER 09; Immorlica, Mahdian SODA 05; Dubins, Freedman AMM 81, Roth MOR 82

Experiments: Che, Sönmez JET 06; Featherstone, Niederle 08; Echenique, Wilson, Yariv 09

Alternative mechanisms: Abdulkadiroğlu, Che, Yasuda AER forthcoming; Kesten QJE 10

Continuum matching models: Abdulkadiroğlu, Che, Yasuda 09; Miralles 08

Example

3 schools A, B, C with capacities: $q_A = q_B = 1, q_C = \infty$
3 students: α, β, γ with preferences and priorities:

Student	Preferences	Priority at
α	$A \succ B \succ C$	B
β	$B \succ A \succ C$	A
γ	$A \succ C$	$-$

Current:

$$\alpha \rightarrow \tfrac{1}{2}A,\ \tfrac{1}{2}B$$
$$\beta \rightarrow \tfrac{1}{2}B,\ \tfrac{1}{2}A$$
$$\gamma \rightarrow C$$

Optimal:

$$\alpha \rightarrow A$$
$$\beta \rightarrow B$$
$$\gamma \rightarrow C$$

Improving on the current mechanism

Any efficient Pareto improvement upon the current mechanism (DA-STB) can be done by the SIC/SOSM algorithm:

- Run DA-STB
- *Trade*: Implement a Pareto improving trade that does not violate stability (a stable improvement cycle)

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- Will students manipulate?
- Impact on welfare?

School Choice

"Nothing is yet known about what kinds of preferences one could expect to be strategically submitted to such a mechanism, or what their welfare consequences would be. Consequently, there is room for more work to further illuminate the tradeoff between efficiency and strategy-proofness."

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⇒ Continuum matching model with cutoffs
(Azevedo, Leshno 2010)

Cutoffs

cutoffs give a tractable representation of matchings

- Cutoff $p \in \mathbb{R}_+^{\mathcal{S}}$ specifies a threshold for every school
- The demand of a student is his most preferred attainable school
- For a school A the aggregate demand $D_A(p)$ is the mass of students that demand A
- p is a *market clearing cutoff* if demand clears:

$$D_A(p) \leq q_A$$

for all $A \in \mathcal{S}$, and $p_A = 0$ if the inequality is strict

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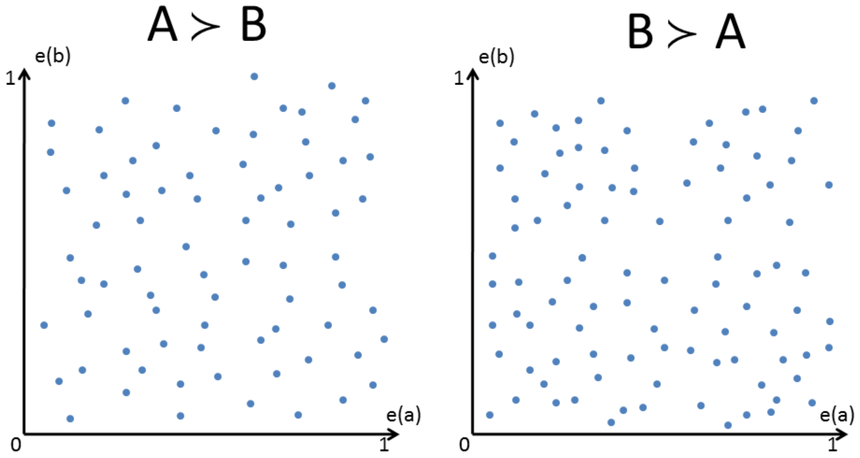
Lemma ($\mu \equiv p$)

(i) *The demand given market clearing cutoffs is a stable matching.*

(ii) *Every stable matching is the demand of some market clearing cutoffs.*

Cutoffs



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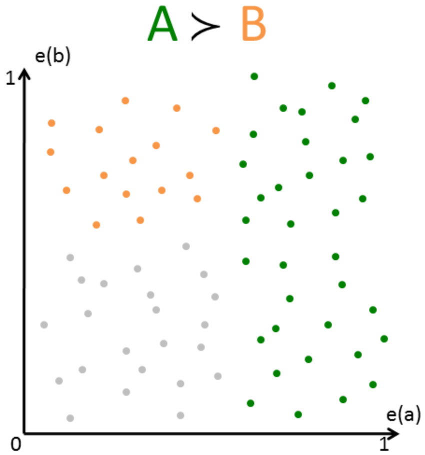

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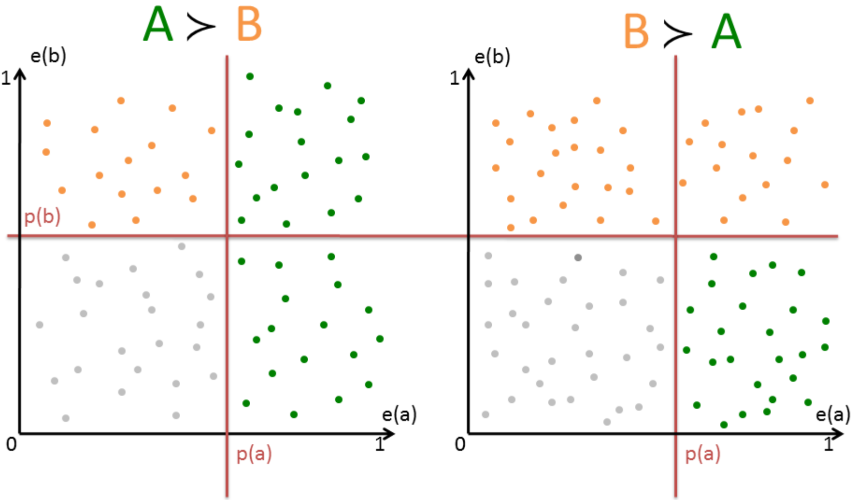

Ex: Trade Manipulations

Two special schools: A = Art , S = Science
Capacities: $q_A = 1$, $q_S = 2$

Student mass $m(\alpha) = m(\zeta) = 1$ and $m(\gamma_A) = 2$
A mass $0 \leq v \leq 2$ of γ_A misreports γ_{AS}:

Type	Preferences	Priority at	Mass
α	$S \succ A$	A	1
ζ	$S \succ A$	–	1
γ_A	A	–	$2 - v$
γ_{AS}	$A \succ S$	–	v

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- α and ζ report truthfully in equilibrium
- γ_A is an EU-maximizer with

$$u_{\gamma_A}(A) = 1 > u_{\gamma_A}(\phi) = 0 > u_{\gamma_A}(S) \geq -0.5$$

Ex: Trade Manipulations

We solve for the DA-STB allocation using the market clearing equations:

$$\begin{aligned} m(\alpha) \cdot p_S + m(\zeta)(p_S - p_A)^+ + m(\gamma_{AS})(1 - p_A) + m(\gamma_A)(1 - p_A) &= q_A \\ m(\alpha)(1 - p_S) + m(\zeta)(1 - p_S) + m(\gamma_{AS})(p_A - p_S)^+ &= q_S \end{aligned}$$

The unique market clearing cutoffs:

$$p_A = \frac{v+2}{v+4}, p_S = \frac{v}{v+4}$$

Ex: Pareto Inefficiency

The DA-STB allocation is:

$$\begin{aligned}
\mu_{DA-STB} : \alpha &\rightarrow \frac{4}{v+4} S, \frac{v}{v+4} A \\
\zeta &\rightarrow \frac{4}{v+4} S, \frac{v}{v+4} \phi \\
\gamma_A &\rightarrow \frac{2}{v+4} A, \frac{v+2}{v+4} \phi \\
\gamma_{AS} &\rightarrow \frac{2}{v+4} A, \frac{2}{v+4} S, \frac{v}{v+4} \phi
\end{aligned}$$

Ex: Pareto Inefficiency

α, γ_{AS} trade to produce the unique SOSM assignment:

$$\begin{aligned}
\mu_* : \alpha &\rightarrow S \\
\zeta &\rightarrow \frac{4}{v+4}S, \frac{v}{v+4}\phi \\
\gamma_A &\rightarrow \frac{2}{v+4}A, \frac{v+2}{v+4}\phi \\
\gamma_{AS} &\rightarrow \frac{3}{v+4}A, \frac{1}{v+4}S, \frac{v}{v+4}\phi
\end{aligned}$$

Ex: Trade Manipulations

For any mass v that manipulates:

- if trade is allowed a γ student that got S has a 50% chance to trade it for A
- γ prefers the lottery $\frac{1}{2}S, \frac{1}{2}A$ over ϕ for sure

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Simple manipulation, works in a large market

Ex: Trade Manipulations

Under DA-STB equilibrium students report truthfully ($v = 0$)

Under the unique trade equilibrium all γ misreport ($v = 2$)

$$\mu_{DA-STB}: \quad \begin{array}{rcl} \alpha & \rightarrow & S \\ \zeta & \rightarrow & S \\ \gamma_A[\gamma_A] & \rightarrow & \frac{1}{2}A,\ \frac{1}{2}\phi \end{array}$$

$$\mu_*: \quad \begin{array}{rcl} \alpha & \rightarrow & S \\ \zeta & \rightarrow & \frac{2}{3}S,\ \frac{1}{3}\phi \\ \gamma_A[\gamma_{AS}] & \rightarrow & \frac{1}{2}A,\ \frac{1}{3}\phi,\ \frac{1}{6}S \end{array}$$

Trade Manipulations

- Manipulations are simple: rank a school in order to trade it
- Students should rank schools by their *trade value*
  - The DA outcome is not the assignment, but an endowment for trade
  - If trade is random a DA-assignment implies a lottery over final assignments
  - Students submit true preferences to DA, but according to trade values
- work even in a large market, rely only on aggregate information

Trade Manipulations

Important welfare implications

- schools valued by the expected trades they command
- students will want to ”grab” over-demanded schools
- trade fails to repair all inefficiencies (some students get stuck)

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- trade fails to repair all inefficiencies (some students get stuck)

$\Rightarrow$ Pareto inefficient wrt to the true preferences

$\Rightarrow$ unstable (IR violations) wrt to the true preferences

Summary

Student optimal stable mechanisms:

1. Can produce unstable outcomes
2. Can:
   - Pareto improve upon DA-STB
   - Be Pareto dominated by DA-STB
   - Be not Pareto comparable to DA-STB
3. Susceptible to (simple) manipulations
4. Manipulable in large markets

- Cutoffs can be useful for analyzing matching problems