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The Welfare Economic Theory of Green National Accounts

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Abstract This paper takes a critical look at the literature on green national accounts. The problem studied is to find a linear index of economic variables that responds to perturbations in the same direction as social well-being. The thesis of a large literature, that net national product (which is a flow) is that index in closed economies, is shown in some interpretations to be simply false and in others to suffer from deep estimation problems. It is argued that capital depreciation using shadow prices should certainly be included in national accounts, but that the right welfare index is a comprehensive measure of wealth, defined as the shadow value of an economy's stock of all capital assets. It is shown that comprehensive wealth is usable as a criterion for policy evaluation as well as for determining sustainable economic development.

Keywords Natural capital · Ecosystems · Capital depreciation · Shadow prices · Income · Comprehensive wealth · Net national product · Intergenerational well-being · Sustainable development · Population growth · Technological uncertainty · Wiener process

1 Introduction: Economic Indices and Their Purposes

In judging the performance of an economy, we usually ask a combination of four questions:

In preparing this review article, I have borrowed extensively from my joint work with Kenneth Arrow, Lawrence Goulder, Karl-Göran Mäler, Kevin Mumford, and Kirsten Oleson. Discussions with them over the years has greatly influenced my thinking on the subject. Some of the exposition has been taken directly from the theoretical Sections in Arrow et al. (2008). I am most grateful to Geir Asheim for his comments on a previous draft of this paper.

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(A) How is the economy doing now? (B) How has its performance been in recent years? (C) How is it likely to perform under alternative policies? (D) What policies should be pursued there?[1]

Public economists address questions (C) and (D); students of the idea of sustainable development are concerned with (B), (C), and (D); while national income statisticians respond to (A). This classification of interests among the three groups of social scientists is no doubt crude, but it helps to dissect a large and varied literature that has studied ways to give quantitative expression to the idea of intergenerational well-being (*well-being*, for short).

Answering questions (C) and (D) requires someone to evaluate an economy at a point in time before and after a (hypothetical) perturbation has been made to it. In contrast, (B) requires the analyst to evaluate when the perturbation is the passage of time itself. However, in order to evaluate an economy, we need a criterion with which to do the evaluation. We also need to justify the choice of that criterion in ethical terms. Interestingly, though perhaps not surprisingly, the same ethical calculus is involved in answers to each of questions (B), (C), and (D).[2] We confirm that below.

Although it is now customary to regard changes in gross national product (GNP) as the criterion for judging the progress or regress of nations, economists have recognized for a long time that, as a measure of well-being, GNP suffers from serious limitations. I review several limitations in Sect. 2 and conclude, as so many economist have already done, that GNP's main weakness lies in the fact that it is insensitive to the depreciation of capital assets. It is then but natural to ask whether net national product (NNP) can serve as a measure of well-being. In Sect. 3 I review the classical argument for NNP (or "net income"), which was that it is the ceiling to be imposed on aggregate consumption if the economy's wealth is not to decline. That argument was put forward by Lindahl (1933) and explored further by Solow (1974), Hartwick (1977), and Dixit et al. (1980). One of the conclusions I reach in this paper (Sect. 4) is that, a huge contemporary literature on "green NNP" notwithstanding, there is nothing more of practical importance to say about NNP's welfare properties than was uncovered by Lindahl.

In tracking the effect of perturbations on well-being, it is useful to make use of shadow prices. That mode of analysis, which is now very well known, is developed in Sect. 4. In Sects. 4.6 and 5–6 I show that shadow prices can be used as relative weights to construct a *linear* index of an economy's stock of capital assets. That index, which we will call "comprehensive wealth", moves in unison with well-being, in that, a perturbation increases (resp., decreases) well-being if and only if, holding shadow prices constant, it increases (resp., decreases) comprehensive wealth (Propositions 4 and 8, below). To be sure, comprehensive wealth isn't the same as well-being, but it responds in the same direction to perturbations as well-being does. As comprehensive wealth is a linear index of the stocks of the economy's (comprehensive) list of capital assets, while well-being is a *non-linear* function of its determinants, it is a far more convenient index to use for responding to questions (B)–(D).[3] I then show (Proposition 9) that, holding shadow prices constant, comprehensive wealth increases

[1] What we mean by an "economy" depends, of course, on the context in which the questions are asked. The economy could be a person, a household, a village or town or district or state or country or region, or even the whole world. Here I assume, for concreteness, that the economy is a country.

[2] See Dasgupta (2001a).

[3] There is no settled term yet for the linear index that I am calling "comprehensive wealth" here. I am borrowing the term from Arrow et al. (2007), but it has been called "genuine wealth" (World Bank 2006), and also "inclusive wealth" (Dasgupta 2007). It is tempting to call it "green wealth" to indicate that natural capital is included in the measure, but that would be to downgrade such other capital assets as health and education. I am hoping that the term "comprehensive wealth" will prevail, because it is vivid.

(resp., decreases) *if and only if* aggregate consumption in the economy is less than (resp., greater than) NNP. Together, Propositions 8 and 9 show that Erik Lindahl's classic definition of NNP (Lindahl called it "income") is very closely related to the finding that comprehensive wealth is a measure of well-being: *one is dual to the other*. Those results not only confirm that policy evaluation should mean estimating the policy's impact on comprehensive wealth (Propositions 2–4), they also give formal expression to the concept of "sustainable development" (Definition 2 and Proposition 7).

Section 4 reports on an extensive recent literature on the welfare economics of green national accounts that has gone a different route from the one initiated by Lindahl. The idea there has been to find ways to *redefine* NNP so that it becomes a measure of well-being. One redefinition identifies the felicity of consumption with consumption itself, while another replaces shadow prices with price indices constructed out of both shadow prices and the rates at which those prices are forecast to change over time. It is reasonable to ask what purpose such exercises serve in a field of enquiry that was created for application to *practical* problems of welfare accounting, where simplicity should be of the utmost concern. Which is why I ask the question in Sect. 4 and report that the answers that have been given are, to my mind at least, wanting.

There is another reason why the recent literature on the welfare economic theory of green NNP should be found wanting. The literature has with but few exceptions been built on three assumptions: (i) population is constant and the economy experiences no exogenous technological change; (ii) the economy is at a full optimum; and (iii) the commodity transformation possibility set is convex.[4] Assumption (ii) permits the authors to assume not only that all goods and services have markets, but also that market prices are the shadow prices being sought. Some of the findings in the recent literature on green NNP remain valid if (ii) is dropped (see below). But as the basis of a theory of *green* national accounts, where "missing markets" should be a dominant thought in our minds, (ii) is a particularly bad assumption: it encourages one to depend entirely on "market observables" when applying welfare economics to policy issues.

Assumptions (ii) and (iii) have proved to be attractive for another reason. They enable one to use the separating hyperplane theorem in defining shadow prices. They also enable one to offer a definition of (comprehensive) wealth that leads to an index which, in the world we have come to know, is applicable neither in policy evaluation nor in the study of sustainable development. (That index is the sum of the present discounted values of the flow of consumption; see Eq. 28.) The problem is that many of the literature's findings are sensitive to assumptions (i)–(iii).[5] But (i)–(iii) fly in the face of evidence. That (i) and (ii) must be dropped in practical applications is self-evident. But (iii) must be dropped as well. In a comprehensive review of the processes that govern the Earth System, Steffen et al. (2004) have shown that, as a near-universal rule, commodity transformation possibility sets are non-convex because non-linearities are a routine feature of Nature's processes. They summarize their findings (p. 266) in the following terms: "Non-linear behaviour is ubiquitous throughout the Earth System; indeed it is much more difficult to find examples of linear than non-linear processes". Many of Nature's processes are even "complex". We should conclude that the "convexity" assumption about the commodity transformation possibility set should have no place in the welfare economic theory of *green* national accounts.

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4 Weitzman (2003) is a book-length treatment of green NNP, based on assumptions (ii)–(iii). See also the survey article by Heal and Kristrom (2005).

5 Brekke (1994) made a similar observation with regard to assumption (i).

So as to develop a theory of green national accounts that is applicable in the world we have come to know, I follow an approach that assumes *neither* that the economy is at a full welfare optimum *nor* that commodity transformation possibilities define a convex set. In Sects. 4 and 5 it is assumed that population is constant and that the processes the economy is subject to are time-autonomous. In Sect. 6 I extend the analysis of Sect. 5 by dropping those two assumptions. Section 7 concludes and points to the many problems that are now urgently in need of study.

2 GNP and National Income

Question (A) stands apart from questions (B)–(D), at least if conventional practice among national income statisticians is any guide. It is now common to summarize the state of an economy by its gross domestic product (or GDP). For simplicity of exposition, we will consider a closed economy. There, GDP is the same as GNP. In a closed economy, GNP is the value of all the final goods that are produced there. But when a commodity is produced and sold, the price paid for the purchase finds its way into someone's pocket in the economy. So GNP can be measured also by adding up everyone's incomes—wages, salaries, interests, profits, and government income. The sum total of all the incomes is called (*gross*) *national income*. As GNP and gross national income are two sides of the same coin, we use the terms interchangeably in what follows.

The concept of GNP was developed originally for market economies, which is why the values imputed to goods and services were market prices. But by a clever construction of shadow prices, economists have adapted GNP even for economies where much economic activity is undertaken in non-market institutions. The empirical literature on rural household incomes drawn from common property resources in poor countries offers a good illustration of the use of shadow prices (Jodha 1986; Cavendish 2000).

Two points should be noted at once: (1) GNP doesn't measure wealth. GNP is a flow (dollars per year, say), whereas wealth is a stock (dollars—period). Similarly, national income is a flow, not stock.[6] (2) Although it has become a commonplace to regard GNP as a welfare index, it is an aggregate measure of the output of final goods and services, nothing more.

A good history of the concepts of GNP and national income was provided by Richard Stone in his Nobel Lecture, which is available on the home page of the Nobel Foundation.

[6] Heal and Kristrom (2008) refer to the present discounted sum of the flow of an economy's aggregate consumption (which is a stock; see Eq. 25) as "national income". A name is but a name, but the Heal-Kristrom move does little to clarify a large and varied literature on the welfare economics of green national accounts. The move confuses matters especially when it is placed next to Brekke (1994, p. 242), Heal (1998), Heal and Kristrom (2005), and World Bank (2006), who define *wealth* to be what Heal and Kristrom (2008) call national income.

Conflating stocks with flows has become a commonplace. In giving expression to their moral outrage over the enormous inequality in today's world, authors of UNDP (1998, p. 30) wrote: "New estimates show that the world's 225 richest people have a combined wealth of over 1 trillion US dollars, equal to the annual income of the poorest 47% of the world's people (2.5 billion)."

As wealth and income differ in dimension, they can't be compared. The stock has to be converted into an equivalent flow (or vice versa) before comparisons can be made. (The authors of UNDP 1999, repeated the mistake.) If we were to pursue UNDP's reasoning, we could follow the standard practice of converting wealth into a figure for permanent income by using a 5% annual interest rate, that is, divide wealth by 20. When this conversion is made on the data, my calculations, albeit they are very crude, tell me that the world's richest 225 people, having a combined annual income of over 50 billion US dollars, earn more than the combined annual incomes of people in the world's twelve poorest countries, or about 7% of the world's population (385 million). This is still a sobering statistic.

But even a cursory study tells us that GNP rose to prominence during the 1930s, when advanced industrial countries were suffering from a deep economic depression. At that time there was need for an index that reflected aggregate economic activity. GNP filled that need. But, somehow, during the post-War years GNP came to be regarded as a welfare index. That interpretation is now so ingrained in us, that growth in GNP is regarded today as the most significant indicator of economic progress. If someone talks of economic growth, the listener doesn't need to ask, "Growth in what?"; he will know that the speaker means growth in GNP.

However, interpreting national income as a measure of well-being has problems. Three are superficial, but one is deep. Let us begin with the former.

(1) Because a dollar in the hands of the poor is awarded the same weight as a dollar in the hands of the rich, GNP doesn't reflect concerns about income inequality.

There is a simple way out of the problem, though. It is to apply income distributional weights, whereby a dollar's worth of income going to a poor household is awarded a greater weight than a dollar going to a rich household. The resulting GNP, being a weighted sum of household incomes, would reflect distributional concerns.[7]

(2) Following Sen (1987) and Dreze and Sen (1990), Anand and Ravallion (1993) and UNDP (1994, pp. 14–15) have criticized those who regard GNP to be a welfare index on grounds that it is instead a measure of a country's opulence, and they remark that opulence is not the same as well-being.

The criticism is faulty in two ways. First, opulence is a stock concept, and GNP is not a return on any index of opulence that I am aware of. Secondly, and more importantly, it isn't a mistake to seek to measure well-being in terms of an index of opulence. The point isn't that opulence misleads, but rather that we should search for the *right* measure of opulence. Below we show that the right measure of opulence is comprehensive wealth.

(3) An educated population produces greater output (Schultz 1961; Becker 1983) as does a healthy population (Dasgupta and Ray 1986, 1987; Fogel 1994, 2004; Dasgupta 1997). So it would seem that GNP responds to improvements in education and health. It has been countered though that, as they reflect mere instrumental virtues of human capital, GNP doesn't adequately reflect the well-being people *enjoy* from becoming educated (Sen 1987, 1999), or from being in good health (Bauer 1971). The way to meet this particular criticism is the same as the one we noted for meeting problem (1), which is to make use of shadow prices for valuing health and education. Health and education are simultaneously aspects of human well-being and factors that produce human well-being. As each possesses two sets of values, health and education should each be counted twice. UNDP (1990) made a step toward including health and education as aspects of human well-being, by constructing its Human Development Index (HDI), which is an aggregate of GNP per capita, life expectancy at birth, and literacy. However, even while advertising the index vigorously over the years, UNDP has offered no ethical justification for the relative weights they attach to the three components of HDI, nor for why the state of the environment should be missing from it.

The deep problem with GNP lies in the fact that the index mismanages intertemporal issues badly. (We confirm below that HDI suffers from that same weakness.) GNP is the acronym for gross national product. The word "gross" means that GNP ignores the depreciation of capital assets. Among natural resources, that depreciation can range from a full 100% of the services drawn from oil and natural gas, to the depreciation experienced by ecosystems from mismanagement. As environmental resources are particularly vulnerable to depreciation through overuse, serious criticisms of GNP were first made in the context of

[7] Dasgupta et al. (1972) applied this technique in their work on social cost-benefit analysis in poor countries. On the use of distributional weights in national accounts and on their relationship to the shadow prices of goods and services in imperfect economies, see Mirrlees (1969), Sen (1976), and Aronsson and Löfgren (1999).

environmental and natural resource problems.[8] It is certainly possible for a country's overall productive base (i.e., the base upon which all economic activity depends) to grow while its GNP increases, which is no doubt a path of economic development we all would like to follow; but it is also possible for a country's overall productive base to *shrink* during a period when GNP grows.[9] The problem is that no one would notice the shrinking if everyone's eyes were rivetted to GNP. If the economy's overall productive base continues to shrink, economic growth will sooner or later stop and reverse sign. GNP will then decline, as will the standard of living, but no one would suspect that a fall was in store. So, growth in GNP per head can encourage us to think that all is well, when it isn't. In that regard, HDI is no better: it is possible for a country's HDI to increase even while its overall productive base shrinks. This means that HDI too can mislead (for illustrations based on international data, see Dasgupta 2001b).

The results just mentioned show that the enormous empirical literature on the sources of economic growth, admirably surveyed by Barro (1997) and Helpman (2004), misdirects. It seems to me we economists should have tried instead to discover those features of economies that protect and promote growth in comprehensive wealth (suitably adjusted for demographic trends). The equations that define cross-country growth regressions have GNP growth on the left hand side. The welfare economic theory of green national accounts says that GNP growth should be replaced by growth in comprehensive wealth. We should want to know for example, whether, other things being equal, political and civil liberties or trade liberalization or any one of the many other features of economies people take interest in promote the accumulation of comprehensive wealth. To the best of my knowledge, no one has studied such questions empirically.

So as to study a number of substantive issues in what follows, we interpret natural resources in a very broad way. Included on our list of resources are assets that provide the many and varied ecosystem services upon which life is based. We also add minerals and fossil fuels to the list. Frequently though, environmental problems are identified in terms of "pollution", not the depletion of natural resources. In fact environmental pollutants are the reverse side of natural resources. In some cases the emission of pollutants amounts directly to a degradation of ecosystems (the effect of acid rains on forests). In others, it means a reduction in environmental quality (deterioration of water quality), which also amounts to degradation of ecosystems (watersheds). For analytical purposes there is no reason to distinguish resource economics from environmental economics, nor resource management problems from pollution management problems, nor resource accounting from environmental accounting. Roughly speaking, "resources" are "goods", while "pollutants" (the degrader of resources) are "bads". Pollution is the reverse of conservation.[10] Some economists today conflate environmental and resource economics by calling it *ecological economics*.

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[8] See Nordhaus and Tobin (1972), Mäler and Wyzga (1976), Dasgupta and Heal (1979), Hartwick (1990), and Lutz (1993), among others. Hartwick (2000) is a fine theoretical treatise on ways to estimate shadow prices for green national accounts. Mäler (1991), and Weale (1997) provide outlines of a complete system of national accounts inclusive of environmental natural resources. Perrings and Vincent (2003) contains applied studies on the subject.

[9] That both are theoretically possible is easy to demonstrate, and I do that below (Sect. 5). For illustrations of those possibilities using crude data from contemporary national accounts, see Dasgupta (2001b), Dasgupta (2007) and Arrow et al. (2007), Arrow et al. (2008).

[10] This point of view was explored in Dasgupta (1982).

3 Green NNP as Maximum Constant Consumption

As GNP doesn't recognize real capital depreciation, why not deduct depreciation from it and use NNP as the welfare measure? Because this looks like a solution to what I have called the "deep problem" with GNP, an extensive contemporary literature has advocated the use of NNP as a measure of well-being.

It cannot be doubted that real capital depreciation should be included in national accounts (see Repetto et al. 1989, and Vincent et al. 1997a, 1997b, on estimates of the depreciation of forest resources in Indonesia and Malaysia, respectively, and on methods for embedding them in national accounts), but the claim that it is GNP from which that depreciation should be deducted in order to obtain a measure of (intergenerational) well-being requires justification.

Interestingly, a justification was provided long ago. The simplest way to describe is to study the consumption-saving decisions of a dynasty of constant population size, operating in a deterministic, competitive market economy.

Let $t(\geq 0)$ denote time, assumed to be continuous. Suppose $r(>0)$ is the market rate of interest on capital, which means that r is net of capital depreciation. We write $W(t)$ for wealth at t, where $W(t) \geq 0$. Imagine that at $t = 0$ the dynasty owns assets worth $W(0)$ in market value. If $C(t)$ is the dynasty's consumption rate, wealth changes according to the equation,

$$\mathrm{d}W(t)/\mathrm{d}t = rW(t) - C(t). \tag{1}$$

Equation 1 says that if the dynasty is determined that its wealth doesn't ever decline, consumption $(C(t))$ must never exceed income $(rW(t))$. Erik Lindahl's definition of "income", as the *maximum* consumption rate that maintains wealth is a generalization of this observation.[11]

Solow (1974) and Hartwick (1977) deepened the content of Lindahl's "income" when they studied an economy where production involves reproducible capital and exhaustible natural resources. Assuming no technological progress, Solow identified conditions on production possibilities under which consumption can be maintained above zero forever. He, and Hartwick subsequently with greater generality, showed that in order to consume at the *maximum* constant rate, net investment in reproducible capital at each moment would have to equal the value of the rate of depletion of natural resources. This result, which was dubbed Hartwick's Rule in Dixit et al. (1980), formalizes Lindahl's suggestion that an economy's income should be defined as the ceiling that must be placed on consumption if it is required that comprehensive wealth never declines. Of course, if substitution possibilities are limited, it could be that sustainable development simply isn't possible. While making a related point, Asheim (1994, p. 262) put it nicely: "Hartwick's rule characterizes a sustainable development; it is not a prescriptive rule for sustainable development."

In the following two sections I argue that, a huge contemporary literature on green NNP notwithstanding, there is nothing more of practical importance to say about NNP's welfare properties than was unearthed by Lindahl.

4 Welfare Indices: Stocks Versus Flows

In a much noted paper, Weitzman (1976) claimed to show that NNP, measured in shadow prices, is proportional to intergenerational well-being. That claim has been widely accepted

[11] See Lindahl (1933). Although in the English-speaking world Lindahl's notion of income has come to be called "Hicksian" income, priority belongs to Lindahl.

as being true, although at times it has been advanced in a more specialized version, as in (S. Dasgupta et al. 2008, p. 7), who write: "Weitzman . . . proved that the present value of current NNP (held constant) is just equal to the present value of consumption along the optimal growth path for the economy."

Even if the claim in S. Dasgupta et al. (2008) were true, it would be of no interest, because well-being is not the same thing as consumption. If, however, Weitzman's claim were true, NNP would be usable as a welfare measure. It is shown in Sect. 5.1 that, excepting in cases where the direct determinants of well-being are constant over time (a case of no practical interest whatsoever), *neither* claim is true. I develop Weitzman's argument in the context of an aggregate model of the economy. The model incorporates the concerns many people today express about the nature of economic progress in a world with depletable natural resources. The results we obtain are self-evidently valid for disaggregated production structures, involving intermediate goods. But concreteness has its uses: it reminds us that the motivation behind the welfare economic theory of green accounting should be *practical* application.

4.1 A Simple Model Economy

At time t, let $K(t)$ be a numerical index of the economy's set of reproducible capital assets (buildings, roads, machines) and let $L(t)$ be the quantity of human capital (health and education) per person. For simplicity, we imagine a population of identical individuals. Population is assumed to remain constant, which we normalize to be 1. So $L(t)$ is the total quantity of human capital at time t.

There are many ways in which Nature enters human activities. Habitat destruction, occasioned by the transformation of natural ecosystems into bricks, glass, steel, and concrete, would be one way to model environmental concerns. Dobson et al. (1997), among others, have constructed such a model. In contrast, I consider the case where Nature's services are extracted for use in production. So let $R(t)$ be a numerical index of those extraction rates. We could imagine that R is measured in biomass units.

Let $A(t)$ be a combined index of publicly known ideas and the effectiveness of the economy's institutions. We interpret $A(t)$ to be *total factor productivity* at t. Now let $Y(t)$ denote the economy's aggregate output, or GNP. We express $Y(t)$ by means of the production function

$$Y(t) = A(t)F(K(t), L(t), R(t)), \tag{2}$$

where F is a non-decreasing, twice differentiable function of each of its arguments, and $F = 0$ if any of its arguments is zero. *I do not assume that F is concave.*

We now describe the evolution of the economy over time. Reproducible capital depreciates at the rate $\lambda(>0)$. Out of GNP, the amount consumed is $C(t)$ and the amount spent in the accumulation of human capital is $J(t)$. $C(t)$ is an aggregate of the rates of consumption of many goods and services, including environmental services. In measuring C, Nordhaus and Tobin (1972) added leisure, direct consumption services from the natural environment, and "negative" consumptions arising from pollution to the list of marketed consumption goods and services. In contrast, the literature on green national accounts being reviewed here has focussed on the amendments that need to be made for estimating investment. Here I follow the latter literature; which is why I introduce aggregate consumption in the model without specifying how that aggregation is achieved. None of the results that follow depends on the way consumption is aggregated.

We don't need to suppose that *ex-ante* and *ex-post* savings are equal. Whether they are or not, the net accumulation of reproducible capital, *ex-post*, satisfies the equation,

$$\mathrm{d}K(t)/\mathrm{d}t = A(t)F(K(t), L(t), R(t)) - J(t) - C(t) - \lambda K_t. \tag{3}$$

As people die, human capital depreciates. Let $\mu(> 0)$ be the proportional rate at which L depreciates. It follows that

$$\mathrm{d}L(t)/\mathrm{d}t = J(t) - \mu L(t). \tag{4}$$

It remains to describe the evolution of natural resources. Let $N(t)$ be the (aggregate) stock of *natural capital* and $M(N)$ the natural growth rate. The dynamics of natural capital is then

$$\mathrm{d}N(t)/\mathrm{d}t = M(N(t)) - R(t). \tag{5}$$

A simple way to think about the dynamics of natural capital is to suppose that $M(N)$ is quadratic. Here we introduce thresholds in the productivity of natural capital. Let m, b, and Q be positive constants. We assume that

$$\begin{array}{ll} M(N(t)) = -b + mN(t)\left[1 - N(t)/Q\right], & \text{for } N(t) > 0, \quad \text{and} \\ M(N(t)) = 0, & \text{for } N(t) = 0 \end{array}. \tag{6}$$

As Nature is productive, we must assume too that $Q > 4b/m$ (otherwise, the right hand side of the first part of Eq. 6 isn't positive for any N). Maximum sustainable yield from Nature is $(mQ/4 - b) > 0$. Moreover, Nature's "carrying capacity" is $Q[1 + (1 - 4b/mQ)^{1/2}]$ and its "threshold" is $Q[1 - (1 - 4b/mQ)^{1/2}]$. If N were to decline to the threshold level, the economy would be doomed (as $F(K, P, 0) = 0$, output would eventually decline to zero). Note that $M(N)$ in Expression 6 is not a concave function.[12]

Modern theories of economic growth assume that total factor productivity A increases with investment in research and development. In some models A is taken to be an increasing function of cumulative output (a case of "learning by doing"). For our purposes there would be no advantages in making any of the assumptions particular to "endogenous" growth theory. So I take it that $A(t)$ is exogenous.

The *state of the economy* is defined by the vector (K, L, N). We write $\underline{S} = (K, L, N)$. Assume that *felicity* (instantaneous well-being) at t depends solely on consumption at t. Writing U for felicity, we have $U(t) = U(C(t))$. We suppose that $U'(C) > 0$ and $U''(C) < 0$ (where $U'(C)$ and $U''(C)$ are the first two derivatives of U with respect to C, respectively). The assumption, $U''(C) < 0$, could appear innocuous, even self-evidently true in welfare economics, but we see below that it became customary in the literature on green national accounts that followed Weitzman (1976), to drop it and assume instead that $U''(C) = 0$.

That U is a function only of C rules out any influence on felicity of habitual consumption or the average consumption of a person's peer group.[13] Moreover, the specification assumes that felicity doesn't depend explicitly on natural capital as stocks (e.g. sacred groves or places of scenic beauty). To introduce such factors in the felicity function would leave the

12 The specification in Eq. 6 is taken from Dasgupta (1982).

13 Intergenerational welfare economics when habitual consumption is an argument of felicity has been studied by Ryder and Heal (1973) and Aronsson and Löfgren (2008). On the welfare implications of the influence of the consumption of others on an individual's felicity, see Layard (1980, 2005) and Arrow and Dasgupta (2007), among others.

propositions we prove below unaffected; it would only affect the structure of shadow prices (Equations 10a–c below).[14]

Let $V(t)$ denote intergenerational well-being at t. $V(t)$ is taken to be of the form

$$V(t) = \int_t^\infty \left[U(C(\tau))e^{-\delta(\tau-t)} \right] d\tau, \tag{7}$$

where $\delta(>0)$ is the rate of time discount. We assume that the integral exists. Expression 7 gives us an ethical calculus with which we address questions (A)–(D).

The expression for V in Eq. 7 has a strong structure. A more general form for V would have a recursive structure, now familiar in contemporary macroeconomics. However, the Propositions on sustainable development that we prove below (Propositions 7–11) would remain unchanged if we were to work with a more general, recursive structure than Expression 7. Because of its familiarity, we shall restrict our discussion to the case where V satisfies Eq. 7.

A now-enormous literature has constructed the welfare economics of green national accounts in an economy at a full optimum. Moreover, the literature has been built on the backs of economic models in which non-convexities in commodity transformation possibilities are out of sight.[15] So as to develop a theory of green national accounts that is applicable, I report an approach that assumes *neither* that the economy is at a full welfare optimum *nor* that production possibilities define a convex set. The approach, which has been developed with increasing generality in Dasgupta and Mäler (2000) and Arrow et al. (2003a,b), bases the welfare economics of green national accounts on *economic forecasts*.[16] It is assumed that the forecast is no mere guesswork, but is based on a theory about the economy (which no doubt has guesswork built into it). This means that the evaluator is able to prepare a forecast not only on the basis of the state the economy happens to be at today, but also from states at which it could conceivably have been today.

4.2 Resource Allocation Mechanisms[17]

Imagine that the economy faces not only technological and ecological constraints, but also institutional constraints (sometimes called transaction and information constraints). By the economy's institutions I mean market structures, the structure of property rights, tax rates, non-market institutions (for credit, insurance, and common property resources), the character of various levels of government, and so forth. We do not assume that the government is bent on maximizing V subject to constraints. It could be that the government is predatory, or is at best neglectful, and has objectives of its own that are not congruent with intergenerational well-being. Nor do we assume institutions to be unchanging over time. What I do assume is that institutions co-evolve with the state of the economy in ways that are considered by the evaluator. It is no doubt a truism that social and political institutions influence the evolution of the state of an economy, but it has also been argued that the state of an economy influences

[14] Such techniques as those that measure "contingent values" (CVM), travel cost, and "hedonic prices", were developed in order that the shadow prices of amenities could be estimated. Freeman (1993) is an excellent treatise on the subject.

[15] For book-length treatments, see Heal (1998), Hartwick (1990), and Weitzman (2003).

[16] Dasgupta (2001b) provides a book-length treatment of the approach, including a very crude attempt to apply it to data from a selected set of countries. Arrow et al. (2004, 2007, 2008) have obtained improved (but still woefully crude!) estimates of sustainable development from recent history.

[17] Here I draw on Dasgupta and Mäler (2000).

the evolution of social and political institutions (Lipset 1959; Acemoglu and Robinson 2006). The theory developed below accommodates this mutual influence.

We study the economy at $t (\geq 0)$. Let $\tau \geq t$. The state of the economy at t is $\underline{S}(t) = (K(t), L(t), N(t))$. Let $\{\underline{E}(\tau)\}_t^\infty \equiv \{C(\tau), R(\tau), J(\tau), K(\tau), L(\tau), N(\tau)\}_t^\infty$ denote an *economic programme* satisfying Eqs. 3–6 from t to ∞, with $\underline{S}(t)$ as the "initial condition".

Definition 1 A resource allocation mechanism, α, is a many-one mapping

$$\alpha : \{\underline{S}(t), t\} \rightarrow \{\underline{E}(\tau)\}_t^\infty. \tag{8}$$

We are *not* assuming that α maps $\{\underline{S}(t), t\}$ into a full optimum at t, nor even that it maps $\{S(t), t\}$ into an efficient programme at t. It may be that α is riddled with economic distortions and inequities. We note again that if institutions and the state of the economy were known to co-evolve, that co-evolution would be reflected in α. Note finally that in our model economy commodity transformation possibilities do *not* define a convex set.

Taking α as a given, Eq. 7 can be written as

$$V(\underline{S}(t), t) \equiv \int_t^\infty \left[U(C(\underline{S}(t), \tau))\mathrm{e}^{-\delta(\tau - t)} \right] \mathrm{d}\tau. \tag{9}$$

$V(\underline{S}(t), t)$ is the *value function* at t. In Expression 9, V is assumed to depend explicitly on t because we allow for the possibility that the economy experiences changes that are exogenous to it (e.g., that $A(t)$ is not constant), a matter to which we return below.

Before putting the concept of resource allocation mechanism to work, it is as well to discuss examples. Imagine first that all capital assets are private property and that there is a complete set of competitive forward markets capable of sustaining a unique equilibrium. In our model here, α would map $\{\underline{S}(t), t\}$ into the full optimum (earlier we assumed, for expositional ease, that households are identical). Much modern macroeconomics is founded on this mechanism, as are many writings on the welfare economics of the environment.[18]

Of particular interest are situations where some of the assets are not private property. Consider the case where K and L are private property, but N is common property. It may be that N is a local common property resource, not open to outsiders. If assets are managed efficiently, we are back to the case of an optimum allocation, albeit one not entirely supported by market prices, but in part by, say, social norms.

On the other hand, it may be that local institutions are not functioning well (e.g., because social norms are breaking down and private benefits from using environmental natural resources exceed social benefits). Suppose in addition that decisions bearing on the accumulation of K and L are guided by the profit motive. Then these behavioural rules together help to determine α. In a similar manner, we could characterize α for the case where environmental natural resources are open to free access (Arrow et al. 2003a).

Assumptions about institutions are embedded in the notion of "resource allocation mechanism". Aspects of the concept of "social capital" (Putnam et al. 1993) appear in our framework as part of the defining characteristics of α, as do ideas relating to "social capability" (Adelman and Morris 1965; Abramovitz 1986), and "social infrastructure" (Hall and Jones 1999). The prevalence (or absence) of trust and honesty are embodied in α, whereas other aspects of the concept of social capital such as personal networks enter as components of human capital (Dasgupta 2000).

18 Heal (1998) contains an account of the latter literature.

4.3 Differentiability of the Value Function

V is taken to be differentiable. Unaided intuition is of little help in judging whether this is strong assumption. The mathematical properties of V depend on the mathematical properties of α. Problems are compounded because household behaviour and production and substitution possibilities among the factors of production are embedded in α, as is the economy's evolving institutional structure. Moreover, there are no obvious limits to the kinds of institutions one can imagine. In many parts of the world the State has been known to act in bizarre and horrible ways. So one looks at what might be termed "canonical" institutions. Analytically, the most well understood are those that support optimum economic programmes. What do we know about them?[19]

If U is concave and the commodity transformation possibility set is convex, V is concave and is therefore differentiable almost everywhere in each component of $\underline{S}$. This property holds even in those circumstances where optimum programmes are chaotic. Thus, chaotic αs don't rule out differentiable Vs (almost everywhere).[20] If the commodity transformation possibility set is not convex, optimum economic programmes can have jumps at certain points on the space of $\underline{S}$s—known as Skiba points (Brock and Starrett 2003). Skiba (1978) showed that at those values of $\underline{S}$ where V is non-differentiable with respect to $\underline{S}$ (such points are, however, non-generic), V is continuous. But if V possesses right- and left-partial derivatives (and it does in every example I have studied), the analysis that follows goes through. The same could be expected to be true for the case of market economies subject to fixed distortions, such as those considered by Little and Mirrlees (1974) in their treatise on social cost-benefit analysis.

Experience with dynamical systems tells us that if α is non-optimal, V can be discontinuous at certain values of $\underline{S}$. Shadow prices would not be definable at such points (see Eqs. 10a–c below). For those αs that have been studied in the literature, discontinuities are however non-generic. So, unless the economy is by fluke at a point of discontinuity, V would be differentiable within a sufficiently small neighbourhood of the initial state. Now, shadow prices would exist if V possessed right- and left-partial derivatives at points at which it is discontinuous. Moreover, if the location of the points of discontinuity on the space of capital stocks are uncertain and the uncertainty is a smooth probability distribution, the expected value of V would be continuous. It would seem then that the demand that V be differentiable does not rule out much of practical significance. The account that follows is valid for a substantially more general set of environments than is usual in writings on intergenerational welfare economics.

4.4 Shadow Prices

Define

$$p(\underline{S}, t) = \partial V(\underline{S}, t)/\partial K(t), \tag{10a}$$

$$q(\underline{S}, t) = \partial V(\underline{S}, t)/\partial L(t), \quad \text{and} \tag{10b}$$

$$n(\underline{S}, t) = \partial V(\underline{S}, t)/\partial N(t). \tag{10c}$$

[19] Differentiability everywhere is a strong assumption. For practical purposes, however, it would suffice to assume that V is differentiable in $\underline{S}$ almost everywhere. The latter would appear to be a reasonable assumption even when commodity transformation possibility sets display non-convexities. See below in the text.

[20] See Majumdar and Mitra (2000) for a fine account. I am grateful to Mukul Majumdar for discussions on this point.

Expressions 10a–c are the (spot) shadow prices at t of reproducible capital, human capital, and natural capital, respectively. The unit of account in Eqs. 10a–c is felicity at t. So as to develop a system of accounts that is neutral among the factors of production, I shall throughout use felicity as *numeraire*.[21]

Using Eqs. 7 and 10a–c, the dynamics of the shadow prices of capital assets can be shown to be,

$$\begin{aligned}\mathrm{d}p(t)/\mathrm{d}t &= \delta p(t) - U'(C(t))\partial C(t)/\partial K(t) - p(t)\partial(\mathrm{d}K(t)/\mathrm{d}t)/\partial K(t)\\ &\quad - q(t)\partial(\mathrm{d}L(t)/\mathrm{d}t)/\partial L(t) - n(t)\partial(\mathrm{d}N(t)/\mathrm{d}t)/\partial N(t), \qquad (11\mathrm{a})\\ \mathrm{d}q(t)/\mathrm{d}t &= \delta q(t) - U'(C(t))\partial C(t)/\partial L(t) - p(t)\partial(\mathrm{d}K(t)/\mathrm{d}t)/\partial K(t)\\ &\quad - q(t)\partial(\mathrm{d}L(t)/\mathrm{d}t)/\partial L(t) - n(t)\partial(\mathrm{d}N(t)/\mathrm{d}t)/\partial N(t), \quad \text{and} \qquad (11\mathrm{b})\\ \mathrm{d}n(t)/\mathrm{d}t &= \delta n(t) - U'(C(t))\partial C(t)/\partial N(t) - p(t)\partial(\mathrm{d}K(t)/\mathrm{d}t)/\partial K(t)\\ &\quad - q(t)\partial(\mathrm{d}L(t)/\mathrm{d}t)/\partial L(t) - n(t)\partial(\mathrm{d}N(t)/\mathrm{d}t)/\partial N(t).^{22} \qquad (11\mathrm{c})\end{aligned}$$

At the initial date $\underline{S}(0)$ is given. α determines the initial shadow prices: $p(0)$, $q(0)$, and $n(0)$. The dynamical system defined by Eqs. 3–7 and 10a–c to 11a–c then allows one to determine the future history of the economy in terms both of quantities and shadow prices.

For ease of notation we have defined one shadow price for each type of capital asset: "reproducible", "human", and "natural". For imperfect αs, this is an awkward move. Consider for example the type of reproducible capital asset that goes by the name, "lathe". It can be that, when α is imperfect, a lathe deployed in an automobile factory has a different shadow price from that of a lathe deployed in a factory making tractors. By lumping all capital assets into a single aggregate, Eq. 10a misses that fact. When, in Sect. 4.6, we show that the welfare criterion that should be used in policy evaluation is comprehensive wealth, we will find it useful to remember that Eq. 10a–c are overly aggregative and that we need to relax them by defining a separate shadow price for each capital asset, in each economic activity in which the asset is deployed.

Shadow prices equal market prices in those competitive market economies where the set of taxes and subsidies that are required to support a full optimum is in place. In imperfect economies, however, market prices are good approximations for shadow prices in the case of only certain commodities (Little and Mirrlees 1969, 1974; Dasgupta et al. 1972). Environmental natural resources are among the many goods and services whose market prices don't approximate their shadow prices. There is now a large body of applied work that has studied ways to estimate the shadow prices of environmental amenities (Freeman 1993). In comparison, the literature on ways to estimate the shadow prices of ecosystem services remains small.[23]

The methods that have been developed for estimating the shadow prices of human capital are applicable mainly in rich countries with a well developed set of markets. Broadly

[21] Dasgupta et al. (1972) and Little and Mirrlees (1974), respectively, developed their accounts of social cost-benefit analysis with consumption and government income as *numeraire*. Which *numeraire* one chooses is, ultimately, not a matter of principle, but one of practical convenience.

[22] I am most grateful to Geir Asheim for correcting the wrong account of shadow-price dynamics in Dasgupta and Mäler (2000).

[23] Daily (1997) is the classic on the ways in which economic activity makes use of ecosystem services. Daily and Ellison (2002) is an excellent overview of the scientific literature. Among a growing number of valuation studies, see Gren et al. (1994), Barbier and Strand (1998), Chichilnisky and Heal (1998), Turner et al. (2000), Boyer and Polasky (2004), Ricketts et al. (2004), Sekercioglu et al. (2004), Bin and Polasky (2005), Naidoo and Adamowicz (2005), Goldstein et al. (2006), and Turner and Daily (2008).

speaking, economists have used two ways to measure improvements in human capital. To illustrate, consider the value of a marginal increase in life expectancy. Some economists have measured the (social) cost of bringing about that increase, while others have estimated the value of the increase itself. The two would lead to the same number at a full optimum, but not in imperfect economies, where the latter is the right way to go about the matter Eq. 10b. But as it requires one to estimate the value of a statistical life (Viscusi and Aldy 2003), the method has proved to be controversial, especially when deployed for making cross country comparisons. Arrow et al. (2004) used figures for public health expenditure as estimates of the shadow price of health improvements in poor countries, while Arrow et al. (2008) have combined age-specific mortality tables with values of statistical life years so as to estimate the shadow price of increases in life expectancy. The latter estimates have been found to be substantially higher than the former, implying that the economies in question are highly imperfect.

Shadow prices at t depend on the extent to which various capital assets are substitutable for one another, not only at t, but beyond t as well. For example, imagine that past profligacy has meant that the stock of natural capital, N, has come perilously close to Nature's threshold, $Q[1-(1-4b/mQ)^{1/2}]$, (see Expression 6). A small further nudge would tip the economy into a basin of attraction that spells d-o-o-m-s-d-a-y. Now introduce a further dose of realism into the model by acknowledging that the threshold level is not known with certainty and that the extraction rate $R(t)$ will never turn out to be exactly as forecast. The shadow price of natural capital, n, would then be very large, signalling that Nature's scarcity value is huge. Even a small further damage to Nature would be recorded as a large negative number. That large figure would be included in estimates of comprehensive investment (defined formally in Eq. 13). A recent suggestion by Mäler et al. (2007) and Walker et al. (2007) that an ecosystem's *resilience* (which, in the present context, is the distance between N and the threshold, namely, $\{N-Q[1-(1-4b/mQ)^{1/2}]\}$) should be regarded as a capital asset, is an important practical move toward the creation of a system of national accounts that takes Nature's non-linearities seriously.[24]

Shadow prices of private goods can be negative if property rights are dysfunctional, such as those that lead to the tragedy of the commons. All future effects on the economy of changes in the structure of assets are reflected in shadow prices. Shadow prices do an awful lot of work for us. That is why they are useful objects.[25]

4.5 Comprehensive Investment

It has been common in the literature on green national accounts to assume that the resource allocation mechanism, α, is "autonomous", by which we mean that $V(\underline{S}(t), t)$ is *not* an explicit function of t. That would be implied in our model economy if total factor productivity (A) in Expression 2 were a constant. For the moment, let us assume that it is a constant.

[24] See Mäler (2008) for an introduction to the economics of resilience.

[25] Martin Weitzman (in conversation) has made the entirely valid point that the machinery of welfare optimization theory can equally be brought to bear on the formulation I am advancing here. He observed that the economic forecast we are studying in the text can be interpreted to be the optimum economic programme subject to the constraint that the programme is the forecast itself. True enough; although an optimization exercise involving a feasible set that contains a single object cannot invoke the "non-empty interior" assumption in optimization theory. That said, one can ask what purpose would be served by introducing the theory of dynamic optimization when the forecast is available. Why not work round the forecast, as I am doing in the text?

So we have $V(t) = V(\underline{S}(t))$. Differentiating V with respect to t and using Eq. 10a–c yields,

$$\mathrm{d}V(\underline{S}(t))/\mathrm{d}t = p(t)\mathrm{d}K(t)/\mathrm{d}t + q(t)\mathrm{d}L(t)/\mathrm{d}t + n(t)\mathrm{d}N(t)/\mathrm{d}t. \tag{12}$$

Write

$$I(t) = p(t)\mathrm{d}K(t)/\mathrm{d}t + q(t)\mathrm{d}L(t)/\mathrm{d}t + n(t)\mathrm{d}N(t)/\mathrm{d}t. \tag{13}$$

$I(t)$ is *comprehensive investment* at t.

Equations (12–13) give us

$$\mathrm{d}V(\underline{S}(t))/\mathrm{d}t = I(t). \tag{14}$$

So we have

Proposition 1 *Comprehensive investment measures the rate at which intergenerational well-being changes with time.*

Hamilton and Clemens (1999) and Dasgupta and Mäler (2000) noted and stressed that Proposition 1 offers a simple criterion that can be applied by national income statisticians to study "sustainable development", an idea that we study in detail in Sect. 5. I argue in Sect. 6 that extensions of Eq. 14 to cases where α is not autonomous will prove to be fundamental in applied studies of questions (B)–(D).

Comprehensive investment (or *investment*, for short) has a well-known welfare interpretation. Imagine that the vector of capital assets at t is not $\underline{S}(t)$ but $\underline{S}(t) + \Delta\underline{S}(t)$, where Δ is an operator denoting a small difference. In the obvious notation,

$$V(\underline{S}(t) + \Delta\underline{S}(t)) - V(\underline{S}(t)) \approx \int_t^\infty \left[U'(C(\tau))\Delta C(\tau)\mathrm{e}^{-\delta(\tau-t)}\right]\mathrm{d}\tau. \tag{15}$$

Now suppose that at t there is a small change in α which takes the form of an increase in investment, but only for a brief moment, Δt. We write the change in the vector of capital assets at $t + \Delta t$ consequent upon the brief increase in investment as $\Delta\underline{S}(t)$. So $\Delta\underline{S}(t)$ is the consequence of the increase in investment at t, and $(\underline{S}(t) + \Delta\underline{S}(t))$ is the resulting vector of capital assets at $t + \Delta t$. Let Δt tend to zero. From Eq. 15 we obtain

Proposition 2 *Comprehensive investment measures the present discounted value of the changes in consumption brought about by it.*

I am unable to say who first proved Proposition 2. It is implicit in Ramsey (1928), who studied the case where α is the full optimum. Marglin (1963) proved the proposition for a simple imperfect economy and used it to develop a theory of social cost-benefit analysis (Dasgupta et al. 1972). Our formulation here shows that Proposition 2 is very general. It forms the welfare basis for responding to questions (C)–(D). We confirm that next.

4.6 Policy Evaluation

Imagine that even though the government does not optimize, it is able to bring about small changes to the economy by altering the existing resource allocation mechanism, α. The perturbation in question could be small adjustments to the prevailing structure of taxes, it could be alterations to the existing set of property rights, or it could be a small public investment project. Call any such perturbation a *policy reform*.

As all policy reforms involve a change in resource allocations, they can be interpreted as investment projects. So consider an investment project that starts at $t = 0$ and terminates at $t = T$. The project can be viewed as a perturbation to α. Although the project's "formal" life is of length T, its effects are felt indefinitely. We consider projects that are small relative to the size of the economy. For concreteness, suppose that the project's output at $t\varepsilon[0, T]$ is $\Delta Y(t)$ and that $\Delta Y(t)$ is obtained by displacing $\Delta K(t)$ and $\Delta L(t)$ units, respectively, of reproducible capital and human capital from the rest of the economy and by increasing the extraction rate of natural capital by $\Delta R(t)$. How should the project be evaluated?[26]

The project's acceptance would perturb $C(t)$ over the entire future ($t \geq 0$). Let that perturbation be $\Delta C(t)$. The corresponding perturbation to $U(t)$ would be $U'(C(t))\Delta C(t)$. But that perturbation includes not only the direct but also all indirect effects, extending beyond T. It would be tiresome for the project evaluator to estimate $\Delta C(t)$ for every project that came up for consideration. Shadow prices are useful because they enable the evaluator to estimate $\Delta C(t)$ indirectly by valuing (i) the project's output, (ii) the additional extraction of natural capital, and (iii) the reallocations of reproducible and human capital. All shadow prices are estimated along α because the project is small.[27]

It is most unlikely that consumption and the two forms of investment in the economy have the same shadow price in an imperfect economy. Moreover, if the evaluator knows α, she knows the way $\Delta Y(t)$ will be divided into additional consumption and investment in reproducible capital and human capital, respectively.

$U'(C(t))$ is the shadow price of $C(t)$. In Sect. 4.4 we wrote $p(t)$ for the shadow price of reproducible capital. That's far too aggregative for our purpose here. Let $\bar{p}(t)$ be the shadow price of reproducible capital in the activity from which $\Delta K(t)$ is to be displaced for deployment in the project at t; and let $\xi(t)$ be the shadow rental on reproducible capital in that activity. It follows that $\xi(t) = \delta + \lambda - (\mathrm{d}\bar{p}(t)/\mathrm{d}t)/\bar{p}(t))$.[28] Similarly, let $(\bar{q})(t)$ be the shadow price of human capital in the activity from which $\Delta L(t)$ is to be displaced for employment in the project; and let $\theta(t)$ be the shadow wage rate (i.e., the shadow rental on human capital) in that activity. Then $\theta(t) = \delta + \mu - (d(\bar{q})(t)/\mathrm{d}t)/(\bar{q})(t))$. Finally, suppose that $\Delta R(t)$ is to be obtained by additional extraction. From Eqs. 5, 6 and 10c, we know that the shadow price of $\Delta R(t)$ is $n(t)$. Knowing α and the shadow prices, our project evaluator can calculate the shadow price of $Y(t)$, which we denote by $y(t)$. But the contribution the project makes to well-being is $\int_0^\infty \left[U'(C(t))\Delta C(t)\mathrm{e}^{-\delta t}\right]\mathrm{d}t$. We conclude that

$$\int_0^\infty \left[U'(C(t))\Delta C(t)\mathrm{e}^{-\delta t}\right]\mathrm{d}t$$

$$= \int_0^T \left[(y(t)\Delta Y(t) - \theta(t)\Delta L(t) - \xi(t)\Delta K(t) - n(t)\Delta R(t))\mathrm{e}^{-\delta t}\right]\mathrm{d}t. \tag{16}$$

[26] If the project has been designed efficiently, we would have:

$$\Delta Y(t) = (A\partial F/\partial K)\Delta K(t) + (A\partial F/\partial L)\Delta L(t) + (A\partial F/\partial R)\Delta R(t).$$

The analysis that follows in the text does not require the project to have been designed efficiently.

[27] Of course, if the project is large relative to the size of the economy, shadow prices alone would not do, and the evaluator would have to estimate the consumer's surpluses that would result from it. These are familiar matters.

[28] $(\bar{p})(t)$ could be discontinuous at t if $\Delta K(t)$ displaced reproducible capital from a different activity from the one it had been rented in a period that ended at t. If it is, the left- and right-hand derivatives of $(\bar{p})(t)$ would be used on the two sides of t.

The right hand side of Eq. 16 is the present discounted value of social profits from the project. And the left hand side of Eq. 16 equals $\Delta V(0)$, the latter being the change in well-being at $t = 0$ that would be brought about by the project. So, Eq. 16 can be re-expressed as

$$\Delta V(0) = \int_0^T \left[(y(t)\Delta Y(t) - \theta(t)\Delta L(t) - \xi(t)\Delta K(t) - n(t)\Delta R(t))e^{-\delta t}\right] dt. \tag{17}$$

Equation 17 leads to the well-known criterion for project evaluation:

Proposition 3 *A project should be accepted if and only if the present discounted value of its social profits is positive.*

Notice that Propositions 2 and 3 in effect say the same thing.

Define "comprehensive wealth" at t to be the shadow value of the economy's comprehensive list of capital assets (see Eq. 27, below). Our investment project reallocates capital assets. The present discounted value of social profits appearing on the right hand side of Eq. 17 is the change the project would bring about to comprehensive wealth at $t = 0$ by the marginal reallocation of capital assets in the economy. Proposition 3 can therefore be re-stated as

Proposition 4 *A project should be accepted if and only if its acceptance would increase the economy's comprehensive wealth.*

In Sect. 5.1 we show that wealth also plays a key role in the analysis of sustainable development. Policy evaluation and the economics of sustainable development involve the same welfare criterion: comprehensive wealth.

Now imagine for simplicity that the economy is "viable", in the sense that it is feasible for the government to embark on an indefinite sequence of policy reforms, where well-being increases at each step. We would imagine that the sequence converges. In view of the non-convexity of the commodity transformation possibility set, the economy would be expected to move asymptotically to a locally optimum resource allocation mechanism. That increase occurs in *real* time. In contrast, an earlier literature derived *tâtonnemont*-like planning mechanisms (with or without prices as coordinating device) that were assumed to be operating in *imaginary* time (Malinvaud 1967; Heal 1973). The object that changes at each step under the sequence of policy reforms we have formulated here is the resource allocation mechanism; whereas, what changes at each step in *tâtonnemont*-like planning mechanisms is the resource allocation itself.

4.7 Green NNP and the Current-value Hamiltonian

In deriving Eq. 14, the key mathematical object was the *value function*, V. Following Weitzman (1976), the literature on green NNP has instead focussed on the economic system's *current value Hamiltonian*.

Using Eqs. 10a–c, the current-value Hamiltonian associated with α can be expressed as

$$H(t) = U(C(t)) + p(t)dK(t)/dt + q(t)dL(t)/dt + n(t)dN(t)/dt.^{29} \tag{18}$$

Combining Eqs. 13 and 18,

$$H(t) = U(C(t)) + I(t). \tag{19}$$

[29] As elsewhere (barring Sect. 4.6), I have suppressed α because it is a given.

The felicity price of consumption is $U'(C)$. It follows that in felicity units NNP at t (which we write as $NNP(t)$), at least as national income statisticians conceive it, is

$$NNP(t) = U'(C)C(t) + I(t). \tag{20}$$

Equations 19 and 20 say that the current-value Hamiltonian is NNP plus "consumers' surplus". That important difference between the Hamiltonian and NNP has been obscured in the literature that has taken its lead from Weitzman (1976). So it is interesting to speculate why that was allowed to happen.[30]

The current-value Hamiltonian possesses an interesting welfare property. To find it, I follow Weitzman (1976) and extend his analysis to imperfect resource allocation mechanisms.

Integrating Eq. 19 over time and using Eqs. 9 and 14 yields,

$$\begin{aligned}\int_t^\infty \left[H(\tau)e^{-\delta(\tau-t)}\right] d\tau &= \int_t^\infty \left[(U(C(\tau)) + I(\tau))e^{-\delta(\tau-t)}\right] d\tau \\ &= V(t) + \int_t^\infty \left[I(\tau)e^{-\delta(\tau-t)}\right] d\tau \\ &= V(t) + \int_t^\infty \left[dV(\tau)/d\tau)e^{-\delta(\tau-t)}\right] d\tau.\end{aligned} \tag{21}$$

Now integrate the second term on the right hand side of Eq. 21 by parts. That yields

$$\int_t^\infty \left[H(\tau)e^{-\delta(\tau-t)}\right] d\tau = \delta \left\{ \int_t^\infty \left[V(\tau)e^{-\delta(\tau-t)}\right] d\tau \right\}. \tag{22}$$

Because Eq. 22 holds for all t,

$$H(t) = \delta V(t). \tag{23}$$

We have therefore proved.

Proposition 5 *The current-value Hamiltonian is the return on intergenerational well-being.*

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30 The literature following Weitzman (1976) is huge. Hulten (1992), Lutz (1993), Asheim (1994), Asheim (2007), Brekke (1994), Hartwick (1994), and Heal and Kristrom (2005) contain expositions of the Weitzman argument and derive some extensions and note a few of its limitations.
Early criticisms of the practice of interpreting the Hamiltonian as NNP are in Dasgupta and Mäler (1991), Dasgupta and Mäler (2000). Weitzman (2001) notes that the Hamiltonian is NNP plus the sum of consumers' surpluses and then observes that the surplus has to be measured by integrating under the "demand functions". Li and Löfgren (2006) have also noted that the Hamiltonian is NNP plus the sum of consumers' surpluses; but then have re-named it "utility NNP". Hartwick (1994, p. 254) called the Hamiltonian "NNP in utils", having assumed that $U''(C) < 0$. Asheim (1994, p. 260) suggested that "... a natural candidate for a concept of NNP in utility terms is (the Hamiltonian)." And Asheim and Weitzman (2001) say that the "... current-value Hamiltonian is essentially comprehensive NNP expressed in utility units." See also Aronsson and Löfgren (1998) for recommending the use of the Hamiltonian as the basis for green accounting in imperfect economies. Those re-namings, however, don't make the Hamiltonian any easier to estimate in practical applications. In any event, I hazard the guess that, if asked, national income statisticians would interpret NNP in "utils" to be Expression 20 in the text, not Expression 19.

Now, $\delta\{\int_t^{\infty}\left[e^{-\delta(\tau-t)}\right]d\tau\} = 1$. So Eq. 23 can be written as

$$H(t) = \delta\left\{\int_t^{\infty}\left[H(t)e^{-\delta(\tau-t)}\right]d\tau\right\} = \delta V(t),$$

from which,

$$H(t)\left\{\int_t^{\infty}\left[e^{-\delta(\tau-t)}\right]d\tau\right\} = \int_t^{\infty}\left[H(t)e^{-\delta(\tau-t)}\right]d\tau = V(t)$$
$$= \int_t^{\infty}\left[U(C(\tau))e^{-\delta(\tau-t)}\right]d\tau. \tag{24}$$

Equation 24 can be summarized as

Proposition 6 *The current-value Hamiltonian at t of an economic programme $\{\underline{E}(\tau)\}_t^{\infty}$ equals the constant-equivalent felicity associated with $\{\underline{E}(\tau)\}_t^{\infty}$.*

Weitzman (1976) proved Propositions 5 and 6 for optimum economies, while assuming that U is linear (i.e., $U''(C) = 0$) in Expression 7. Now Eqs. 19 and 20 say that if U is linear, the Hamiltonian $H(t)$ equals NNP, $NNP(t)$. So it has become a commonplace to say that $NNP(t)$ is the constant-equivalent felicity at t.

Against all evidence, both from observed human behaviour with regard to consumption smoothing and from ethical concerns over inequality, why did Weitzman (1976) assume $U(C)$ to be a linear function?

Weitzman (1998, pp. 1,583) has offered the following account:

"... it is assumed that, in effect, there is just one composite consumption good. It might be calculated as an index number with given price weights, or as a multiple of some fixed basket of goods, *or, more generally, as a cardinal utility-like aggregator function*. The important thing is that the consumption level in period t can always be registered unambiguously by the single number $C(t)$". (Emphasis added).[31]

Dasgupta and Mäler (1991, 2000) noted a problem with Weitzman's reasoning. Measuring non-linear felicity functions for an entire economy would involve estimating economy-wide consumer surpluses. The practical appeal of such indices as NNP and wealth is that they are linear functions of quantities. We are being asked, however, to abandon practical advantages in order to accommodate a particular interpretation of the Hamiltonian. And we are asked to do it by simply renaming the "felicity of consumption" as "consumption". It would be interesting to know how national income statisticians would react to a proposal that henceforth they should estimate their economy's current-value Hamiltonian.[32]

[31] Earlier, in a very clear discussion of Weitzman (1976), Brekke (1994, p. 242) assumed that "... (shadow) prices are ... normalized so that an extra dollar of consumption at any point in time adds the same amount to utility." Hartwick (1994, p. 254) interpreted Proposition 5 to be saying, "... current NNP(t), in utils, is interest on past accumulations of investment, valued at current prices." By "current NNP(t)" Hartwick meant the current-value Hamiltonian at t.

[32] If U is homogeneous of degree $h(0 < h < 1)$, one can construct a measure that looks like NNP and is proportional to the Hamiltonian. However, it involves using prices that do not reflect social scarcity values. To see how this can be done, notice that for the case in hand, Eq. 19 can be written as

$$H(t) = h\mathrm{d}U(C(t))/\mathrm{d}C(t) + I(t).$$

In response, Weitzman (2000), following a suggestion in Sefton and Weale (1996), has argued that a simple recalibration of U enables one to interpret NNP as the constant-equivalent felicity rate. His argument is as follows:

Recall that the ethical ordering of consumption paths is invariant under positive affine transformations of U. So, if U is a felicity function, one could as well use $(aU + b)$, where a and b are constants, and $a > 0$. This means there are two degrees of freedom when U is calibrated. Let us assume that $U'(C) > 0$ and $U''(C) < 0$, and let $C(0)$ be consumption at $t = 0$. Choose a and b so that $a\mathrm{U}'(C(0)) = 1$ and $(aU(C(0)) + b) = C(0)$. The idea therefore is to so calibrate U that initial felicity equals initial consumption (expressed in felicity *numeraire*) and initial marginal felicity equals one. This makes the Hamiltonian at $t = 0$ equal to NNP at $t = 0$. It follows from Eq. 24 that at $t = 0$, NNP can be interpreted as the constant-equivalent felicity associated with α.

There is a problem though. A high or low value of U in itself carries no significance (a and b are freely chooseable, remember). So, to be told that today's NNP, expressed in felicity *numeraire*, is high (or low) because the constant-equivalent U is high (or low), in itself has no meaning. What would have meaning are *comparisons* of U (equivalently, V); across time, or space, or groups of people. It would certainly be informative if we could be told, say, that because NNP is expected to be greater tomorrow than it is today, tomorrow's constant-equivalent U can be expected to be greater than what it is today. If we were to be told that, we would deduce that V tomorrow should be expected to be higher than V today. Unfortunately, we cannot in general be told that. The reason is that once U has been calibrated at $t = 0$, it must not be recalibrated, ever again. To do so would be to alter the underlying ethical ordering of economic programmes, which would make intertemporal comparisons of V meaningless. But unless U is constant over time (i.e., unless C is constant), it would have to be recalibrated continuously if Weitzman's interpretation of NNP were to be preserved at each date. We should conclude that Propositions 5 and 6 do not offer a normative interpretation of NNP.

There is another, seemingly plausible, way to justify the use of a linear felicity function in discussions of sustainable development. Assume that the commodity transformation possibility set is convex. Assume too that the value function is Expression 7, where $U''(C) < 0$, and that α is the full optimum. Consider an arbitrary date $t \geq 0$. For $\tau \geq t$, let $C^*(\tau)$ maximize $V(t)$ in Expression 7. By the separating hyperplane theorem, among all feasible consumption paths, $C^*(\tau)$ also maximizes

$$V^*(t) = \int_t^\infty \left[U'(C^*(\tau))C(\tau)\mathrm{e}^{-\delta(\tau-t)}\right]\mathrm{d}\tau. \tag{25}$$

In Eq. 25, $V^*(t)$ is linear in $C(\tau)$.

The current-value Hamiltonian corresponding to $V^*(t)$ is

$$H^*(t) = U'(C^*(t))C(t) + I(t), \tag{26}$$

where $I(t)$ is (comprehensive) investment in the "linearized" maximization problem. So $H^*(t)$ can be interpreted as $NNP(t)$ in felicity units.

Footnote 32 continued
Then

$$NNP(t) = H(t)/h = [\mathrm{d}U(C(t))/\mathrm{d}C(t)]\,C(t) + I(t)/h.$$

Notice though that in this expression the prices of investment goods relative to those of consumption goods do not reflect social scarcity values. This is why the result should be of little interest.

That's the argument. The problem with it is that unless $C^*(\tau)$ happens to be constant, $H^*(t)$ is not proportional to $V(t)$. But $C^*(\tau)$ is constant only in a Solow–Hartwick economy, where the government follows the "max–min" consumption policy. In that economy, however, $\mathrm{d}V(t)/\mathrm{d}t = \mathrm{d}V^*(t)/\mathrm{d}t = 0$. Recall also that the argument has required us to assume, against all evidence from the environmental sciences, that commodity transformation possibilities define a convex set. We should conclude once again that NNP does not have the welfare property that has been claimed on its behalf.[33]

5 Sustainable Development and Growth in Comprehensive Wealth: The Autonomous Case

In the concluding paragraph of a famous paper on national income, Samuelson (1961) suggested that perhaps intergenerational well-being comparisons can only be made with some "wealth like concept". The suggestion was tentative, in the form of a remark. Samuelson offered no proof.

Pearce and Atkinson (1993), Hamilton (1994), Pearce et al. (1996), and Serageldin (1995) argued that wealth is the appropriate index to use for judging whether economic development is sustainable. However, they too offered no proof. Using Proposition 1, Hamilton and Clemens (1999) and Dasgupta and Mäler (2000) offered proofs (for the fully optimal and imperfect economies, respectively) that the correct *linear* index of well-being is not NNP (which is a flow), but *comprehensive* wealth (which is a stock) at constant shadow prices. The motivation behind their viewpoint came from a parallel literature in environmental economics. A famous 1987 report by an international commission (widely known as the Brundtland Commission Report) defined *sustainable development* as "... development that meets the needs of the present without compromising the ability of future generations to meet their own needs." (World Commission on Environment and Development 1987). In this reckoning, sustainable development requires that relative to their populations each generation should bequeath to its successor at least as large an overall *productive base* as it had itself inherited. The requirement is derived from a relatively weak notion of intergenerational justice. Sustainable development demands that future generations have no less of the means to meet their needs than we do ourselves; it demands nothing more. But how is a generation to judge whether it is leaving behind an adequate productive base for its successor?[34]

Even though the Brundtland Commission Report was at best hinting at the maintenance of some aggregate index of the *means* of production (what else could "meeting their own needs" mean?), a definition of sustainable development can be obtained directly from the *ends* themselves. Let us first formulate the idea of sustainable development *at a point in time*:

Definition 2 The economic programme $\{\underline{E}(\tau)\}_t^\infty$ reflects sustainable development at t if $\mathrm{d}V(t)/\mathrm{d}t \geq 0$.

In Sect. 5.3 we will introduce a more demanding notion of sustainable development.

[33] Dasgupta and Mäler (1991), Dasgupta and Mäler (2000) showed that NNP is the appropriate criterion for project evaluation provided that projects are small *and are of the briefest duration*. The intuition behind the result is that shadow prices don't change if the period is short. However, as investment projects in practice are not of the briefest of durations, their result is of no interest.

[34] For an excellent taxonomy of various ways of interpreting "sustainable development", see Pezzey (1992). See also Pezzey and Toman (2002).

5.1 Criterion for Sustainable Development at a Moment in Time

If α is autonomous, a simple criterion for sustainable development follows from Definition 2. We can rephrase Proposition 1 as

Proposition 7 *Development is sustained at t if and only if (comprehensive) investment at t is non-negative.*

From Eq. 13 we know that investment is the value of net changes in the economy's (comprehensive) set of capital assets. So we write *comprehensive wealth* as

$$W(t) = p(t)K(t) + q(t)L(t) + n(t)N(t). \tag{27}$$

I shall presently show that Expression 27 is the correct definition of comprehensive wealth (or *wealth*, for short). In contrast, Hulten (1992), Brekke (1994), Heal (1998), Heal and Kristrom (2005), and World Bank (2006), following Fisher (1930), have interpreted wealth at t to be the present discounted value of consumption. To formulate the latter, suppose at t the consumption forecast is $\{C^*(\tau)\}_t^\infty$. Then for any consumption path $\{C(\tau)\}_t^\infty$, Fisherian wealth, which we denote by $W^*(t)$, is

$$W^*(t) = \int_t^\infty \left[U'(C^*(\tau))C(\tau)\mathrm{e}^{-\delta(\tau-t)}\right]\mathrm{d}\tau.^{35} \tag{28}$$

But $W(t) = W^*(t)$ if *and only if* (i) the economy is at a full optimum and (ii) the commodity transformation possibility set is subject to constant returns to scale.[36] In their empirical work on sustainable development in an economy with changing population (Sect. 6.3 below), Dasgupta (2001b) and Arrow et al. (2004, 2007, 2008) found it necessary to estimate the economy's wealth-GNP ratio. They showed that the concept of wealth that was needed for that task is the one formalized in Expression 27. Which is why, as a definition of "wealth", Expression 28 isn't useful. Of course, a name is but a name, and one could choose to call $W^*(t)$ "wealth" and call $W(t)$ by some other name. The point though is that it is $W(t)$ in Expression 27 that emerges as the primary concept in the welfare economics of green national accounts; not Expression 28.

We summarize this in the form of

Definition 3 Comprehensive wealth is the shadow value of the stocks of the economy's (comprehensive) list of capital assets.[37]

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35 As δ is the felicity rate of discount, $\mathrm{e}^{-\delta t}$ is the felicity discount factor that is to be applied to $U(t)$ when the latter is viewed at $t = 0$. So $U'(C^*(t))\mathrm{e}^{-\delta t}$ is the consumption discount factor that should be applied to $C(t)$ when the latter is viewed at $t = 0$.

36 For helpful discussions on this point, I am most grateful to Geir Asheim.

37 Li and Löfgren (2006) define comprehensive wealth to be the value function, Expression 7:

$$V(t) = \int_t^\infty \left[U(C(\tau))\mathrm{e}^{-\delta(\tau-t)}\mathrm{d}\tau\right].$$

Again, a name is but a name; but, as $V(t)$ includes the entire battery of consumer surpluses, the value function suffers from the same practical deficiencies that we earlier detected in the Hamiltonian. In any event, if national income statisticians are by a miracle able to estimate $V(t)$, no further work would be needed on the welfare economics of green national accounts: we should use $V(t)$ directly for both project evaluation and sustainability analysis!

Standard national income accounts suggest that capital-output ratios are typically of the order of 3–4 years. Arrow et al. (2008) have found the ratio of comprehensive wealth to GNP in some countries in their sample to be as large as 10–12 years. And those are substantial under-estimates because the authors were unable to include, among other things, the value of ecosystems in their estimates of the wealth of nations. It would seem that contemporary economic activities involve far greater amounts of "capital" than are recorded in official national accounts.

Wealth (as defined in Expression 27) is a key concept because, using Expression 27 and definition (3), we can rephrase Proposition 5 as

Proposition 8 *Development is sustained at t if and only if, holding shadow prices constant, comprehensive wealth is non-declining at t.*

A simple application of Proposition 8 tells us that it is possible for an economy's GNP to increase for a period along an unsustainable development path. Consider the "cake-eating economy", made famous by Hotelling (1931). The economy's consumption good is a durable, non-deteriorating asset, saving which yields zero return. GNP in consumption units is $C(t)$. Suppose that the resource allocation mechanism, α, is imperfect. Imagine that under α, $C(t)$ is forecast to increase during an initial interval of time, $[0, T]$, followed by ever-decreasing consumption (as is inevitable in the cake-eating economy). If $p(t)$ is the shadow price of the "cake" at t, relative to consumption, then investment is $-p(t)C(t)$. So, GNP is forecast to grow during $[0, T]$, even though wealth is forecast to decline during that same period.

Interestingly, Propositions 7 and 8 give us the sense in which NNP *is* a welfare index. Equation 20 says that NNP is the sum of investment and the value of consumption. So we have,

Proposition 9 $dV(t)/dt \geq 0$ *if and only if* $U'(C(t))C(t) \leq NNP(t)$.

Proposition 9 says that if (intergenerational) well-being is not to decline, the value of consumption services must not exceed NNP. This is the sense in which NNP has normative significance; it is also the sense that was explicit in Lindahl's formulation of the concept of income.

To illustrate Proposition 9, consider once again Hotelling's "cake-eating economy". As is well known, if $\eta(\geq 1)$ is the elasticity of marginal felicity, optimum consumption declines at the rate δ/η. Let $C^*(t)$ be the optimum rate of consumption. Dasgupta and Heal (1979) noted that in felicity units, $GNP(t) = U'(C^*(t))C^*(t)$, although $NNP(t) = 0$ (because investment is $-U'(C^*(t))C^*(t) < 0$). It follows that wealth at t declines at the rate $-U'(C^*(t))C^*(t)$. Moreover, $U'(C^*(t))C^*(t) > NNP(t) = 0$.

An alternative way to characterize sustainable development is to use Eqs. 19 and 23 to obtain

$$\delta\,[\mathrm{d}V(t)/\mathrm{d}t] = U'(C(t))\mathrm{d}C(t)/\mathrm{d}t + \mathrm{d}I(t)/\mathrm{d}t. \tag{29}$$

Recall Eq. 18. We can express Eq. 29 as

Proposition 10 *Development is sustained at t if and only if the value of net changes in the flow of consumption services plus the change in (comprehensive) investment is non-negative.*

This rather elementary result, which appears as Theorem 1 in Brekke (1994) for the fully optimum economy and as Proposition 5 in Dasgupta and Mäler (2000) for imperfect economies, is dependent on the *numeraire* we have chosen, namely felicity. Asheim and Weitzman

(2001) have developed Proposition 10 by identifying suitable price indices (namely, the Divisia index) which, when used in the estimation of NNP, transforms the latter into a welfare measure. The price indices aggregate both shadow prices and their rates of change over time in a non-linear way so as to make Proposition 10 read as saying that $\mathrm{d}V(t)/\mathrm{d}t \geq 0$ if and only if $NNP(t)$, when estimated on the basis of those indices, is non-declining at t (see also Li and Löfgren, 2006).

I have never quite grasped the motivation behind that particular result. Proposition 10 demands everything that needs to be measured in Propositions 7 and 9, but demands a further set of measurements in the form of price changes. In studying sustainable development at a moment in time, it would be a lot simpler to estimate comprehensive investment and confirm its sign.[38]

5.2 Welfare Comparisons Across Space[39]

Cross-country welfare comparisons are a commonplace today. Let us see how they could be conducted when the criterion that is used to compare economies is comprehensive wealth.

It is simplest to consider a continuum of closed economies, parametrized by x (a scalar). We may interpret differences among economies in terms of differences in initial endowments, or behavioural characteristics, or the resource allocation mechanisms guiding them. But in order to make meaningful comparisons of intergenerational well-being, we must be able to ascribe the same felicity function (U) and the same time discount rate (δ) to all economies.

As comparisons among the economies are being made at a given date, we drop the time subscript. Let $V(x)$ be the value function in economy x. It follows that

$$\mathrm{d}V(x)/\mathrm{d}x = p(x)\mathrm{d}K(x)/\mathrm{d}x + q(x)\mathrm{d}L(x)/\mathrm{d}x + n(x)\mathrm{d}N(x)/\mathrm{d}x + \partial V(x)/\partial x. \quad (30)$$

If the resource allocation mechanism is autonomous in location, $\partial V(x)/\partial x = 0$ in Eq. 30. So we have

Proposition 11 *Intergenerational well-being in an economy is higher (respectively, lower) than in any of its immediate neighbours if, it is wealthier (respectively, less wealthy), when evaluated in terms of its shadow prices.*

The assumption that the economies form a continuum allows us to conduct local comparisons. If the economies are finite in number, they must differ by discrete amounts. Cross-economy comparisons now would involve a comparison of consumer surpluses (Weitzman 2001), just as in the evaluation of large investment projects. Below we follow Arrow et al. (2003a) by analyzing the equivalent problem that arises when the idea of sustainable development is extended to cover an interval of time.

5.3 Sustainability over an Interval of Time

Definition 2 offered a local measure of sustainable development. It is useful to derive a non-local measure. The strongest version would be

Definition 4 The economic programme $\{\underline{E}(\tau)\}_t^\infty$ reflects sustainable development if $\mathrm{d}V(t)/\mathrm{d}t \geq 0$ for all t (≥ 0).

[38] In placing that literature in a wider general equilibrium context, Sefton and Weale (2006) and Asheim (2007) offer a taxonomy of the relationships among alternative ways of redefining NNP that make it a welfare index.

[39] This section is taken from Dasgupta and Mäler (2000).

It is an easy matter to derive the counterparts of Propositions 7–11 when "sustainable development" is understood to cover all of future, as in Definition 4. So we ask a somewhat different question: what criterion would be appropriate for assessing sustainable development over an interval of time, $[0, T]$?

We continue to assume that α is autonomous. Write

$$G(0, T) \equiv \int_0^T [(\mathrm{d}p(t)/\mathrm{d}t)K(t) + (\mathrm{d}q(t)/\mathrm{d}t)L(t) + (\mathrm{d}n(t)/\mathrm{d}t)N(t)]\,\mathrm{d}t. \tag{31}$$

Integrating Eq. 14 from 0 to T and using Expression 31 yields

$$V(T) - V(0) = W(T) - W(0) - G(0, T). \tag{32}$$

We may use Eq. 32 to state

Proposition 12 *In order to assess whether intergenerational well-being has increased between two dates, the capital gains on the assets that have accrued over the interval should be deducted from the difference in (comprehensive) wealth between the dates.*

Because the left hand side of Eq. 32 involves a finite difference between intergenerational well-being at two dates, we expect an underlying index number problem to appear. The right hand side of Eq. 32 says that the problem does appear and it offers the solution as well: the integral over time of capital gains enjoyed by the assets (the term, $G(0, T)$) has to be included in welfare comparisons. At a full optimum, the shadow prices would, of course, be market observables (Weitzman 2001 notes this). In imperfect economies, however, they are not market observables, but have to be estimated on the basis of the resource allocation mechanism and the underlying conception of intergenerational well-being.

Propositions 7–11 are equivalence relationships, nothing more. It could be that the economy is incapable of achieving sustainable development indefinitely, owing to scarcity of resources or limited substitution possibilities among capital assets or because the scale of the economy is too large. In which case it wouldn't be possible to accumulate wealth indefinitely. To consider another example, it could be that although the economy is in principle capable of sustainable development over the indefinite future, $V(t)$ declines along the path that has been forecast because of bad government policies; say, because α gives rise to a profligate use of natural resources. Suppose that profligacy gives rise to fast economic growth in the form of a sharp rise in GNP over a period of time. It could be that $V(t)$ increases for a while, but the increase is not permanent. The earlier profligacy could be a reason why shadow prices of natural resources assume such high values in future years as to make it impossible for the economy to undertake positive (comprehensive) investment. Of course, the extent to which the economy would be able to achieve substitution of reproducible capital and human capital for natural capital would depend on a number of factors, including the rate of technological progress. There can even come a point where no amount of investment in K or L is able to offset further declines in N (Ehrlich and Goulder 2007). The age of substitution would then be over, at least until further scientific and technological breakthroughs take place.

To take yet another example, it could be that even when α is optimal, δ has been chosen to be so large that $V(t)$ declines along the optimum economic programme. (It can even be that along an optimum path, $V(t)$ declines for a period and then increases thereafter.) "Sustainability" and "optimality" are different concepts.

6 Sustainable Development and Growth in Comprehensive Wealth: Non-autonomous Cases

The time-autonomous case that we have just studied is useful as a way of fixing our ideas about the welfare economics of green national accounts. In their pioneering empirical study, Hamilton and Clemens (1999) offered estimates of comprehensive investment rates during the period 1970–1993 in over 120 countries. They showed that in the case of many countries the numbers differed considerably from (official) investment figures, inasmuch as the latter didn't take account of (dis)investments in forests, oil and natural gas, and the atmosphere as a sink for carbon emissions. But in order to convert estimates of changes in wealth into figures that would indicate whether a country's economic development had been sustainable, population change needs to be taken into account as well as changes in total factor productivity (the "residual"), among other things. Hamilton and Clemens didn't offer any analysis of how their assessment of the progress and regress of nations during 1970–1993 would be affected if population growth and the residual were taken into account. Which is why the Hamilton-Clemens numbers didn't amount to an empirical enquiry into sustainable development. Dasgupta (2001b), Arrow et al. (2004, 2007, 2008), and World Bank (2006) have attempted to do that. They observed that if we are ever to put the theory to use, there is no escape from cases in which α is non-autonomous. As we see below, those extensions require answers to very difficult questions.

As V is now an explicit function of time, we write $V = V(\underline{S}(t), t)$. Differentiating V with respect to t yields a generalization of Eq. 14:

$$\begin{aligned} \mathrm{d}V(\underline{S}(t))/\mathrm{d}t &= \partial V/\partial t + p(t)\mathrm{d}K(t)/\mathrm{d}t + q(t)\mathrm{d}L(t)/\mathrm{d}t + n(t)\mathrm{d}N(t)/\mathrm{d}t \\ &= \partial V/\partial t + I(t). \end{aligned} \tag{33}$$

An easy way to interpret $\partial V/\partial t$ is to regard time itself as a capital asset, say Z; implying that its accumulation is given by the condition,

$$\mathrm{d}Z/\mathrm{d}t = 1. \tag{34}$$

With Eq. 34 defining the new asset, $\partial V/\partial t (= \partial V/\partial Z)$ becomes the shadow price of time; while the right hand side of Eq. 33 becomes investment inclusive of the accumulation of Z.

The question arises as to why α should be non-autonomous. There could be many reasons. Here I discuss three reasons that were studied in the empirical exercises in Dasgupta (2001b) and Arrow et al. (2004, 2007, 2008), and a fourth reason that has been uncovered in an important and interesting study by Marsili (2008) on the measurement of wealth when future values of total factor productivity are uncertain.

6.1 The Residual

α would be non-autonomous if certain types of technological and/or institutional change are exogenous. In our model economy, V would be an explicit function of time if the "residual", $[\mathrm{d}A(t)/\mathrm{d}t]/A(t)$, is not zero. It can be argued though that if every factor of production could be accounted for, every quality change in goods and services measured, and every institutional change traced to the dynamics of capital accumulation and decumulation, the residual would be zero in a closed economy. To deny that would be to imagine that there is "free lunch". But there will always be factors of production that escape the notice of accountants. Nor do political scientists have a tight enough theory of endogenous institutions. So we should not expect the residual to be zero even in closed economies.

In open economies there are further reasons why the residual may not be zero. It is possible for countries to obtain knowledge from elsewhere for free. The growth of such knowledge in the producing countries may be endogenous, but it would not be endogenous for those that are obtaining the knowledge without effort. In their empirical work on sustainable development, Arrow et al. (2004, 2007, 2008) identified conditions under which $\partial V/\partial t$ is proportional to the residual. They then used Eq. 33 to explore whether development has been sustainable in recent years in a selected set of countries.

That most environmental resources go unrecorded in growth accounting has an important implication for estimates of the residual. Denison (1962) argued that if capital depreciation is not included when the residual is estimated, the latter would be higher than its true value. It follows that if the use of natural capital in an economy has in fact been increasing, but not recorded, estimates of the residual would be biased upward. Tzouvelekas et al. (2006) have shown that the bias can not only be significant, but that it may be that the true residual is nil even though the estimated residual is positive.

6.2 Global Public Goods

Countries interact with one another not only through trade in international markets, but also via transnational externalities. In estimating comprehensive investment in a sample of 120 countries, Hamilton and Clemens (1999) added carbon dioxide in the atmosphere to the list of capital assets, but regarded the shadow price (a negative number) of a country's emission to be the sum of the shadow prices of all countries. Their procedure would be valid if each country were engaged in maximizing global welfare, an unrealistic scenario. We now develop the required analysis for global public goods generally.

Let $G(t)$ be the stock of a global public good at t. We may imagine that G is measured in terms of a "quality" index which, to fix ideas, we shall regard as carbon dioxide concentration in the atmosphere. Being a global public good, G is an argument in the V function of every country. Let $\underline{S}_j(t)$ be the vector of assets owned by residents of country j. Let α_j be j's resource allocation mechanism and $\underline{\alpha}$ the vector of resource allocation mechanisms. If V_j is intergenerational well-being in j, we have, in the obvious notation,

$$V_j(t) = V_j(\alpha_j, \underline{S}_j(t), G(t)).$$

Reverting to utility numeraire for expositional ease, let $p_j(t)$, $q_j(t)$, and $n_j(t)$ be the shadow prices in country j of reproducible capital, human capital, and natural capital, respectively, and $g_j(t) = \partial V_j(t)/\partial G(t)$. It may be that G is an economic "good" for some countries, while it is an economic "bad" for others. For the former, $g_j > 0$; for the latter, $g_j < 0$. Let $E_j(t)$ be the net emission rate from country j and $E(t)$, the net aggregate emission. It follows that

$$\mathrm{d}G(t)/\mathrm{d}t = \sum_j (E_j(t)) = E(t). \tag{35}$$

In order to focus on investment in the global public good, write

$$I_j(t) = p_j(t)\mathrm{d}K_j(t)/\mathrm{d}t + q_j(t)\mathrm{d}L_j(t)/\mathrm{d}t + n_j(t)\mathrm{d}N_j(t)/\mathrm{d}t. \tag{36}$$

Using Eq. 33, Eq. 36 generalizes to

$$\mathrm{d}V_j(t)/\mathrm{d}t = \partial V_j/\partial t + I_j(t) + g_j(t)\mathrm{d}G(t)/\mathrm{d}t. \tag{37}$$

On using Eqs. 35, 37 becomes

$$\mathrm{d}V_j(t)/\mathrm{d}t = \partial V_j/\partial t + I_j(t) + g_j(t)\sum_k (E_k(t)). \tag{38}$$

Notice that the expression on the right hand side of Eq. 38 does not depend on whether $\underline{\alpha}$ is based on international cooperation. On the other hand, $\mathrm{d}K_j(t)/\mathrm{d}t$, $\mathrm{d}L_j(t)/\mathrm{d}t$, $\mathrm{d}N_j(t)/\mathrm{d}t$, and $\mathrm{d}G(t)/\mathrm{d}t$ *do* depend on international resource allocation mechanisms (e.g., whether the countries cooperate) and they affect all the shadow prices. Hamilton and Clemens (1999) identified the "net benefit" to country j from emissions as $(\Sigma_k g_k(t))E_j(t)$, whereas, as Eq. 38 shows, the correct formula is $g_j(t)[\Sigma_k E_k(t)]$. If countries act in their own interest, the two expressions are equal only under very special circumstances (e.g., if the countries were identical).

Arrow et al. (2008) have estimated the term $g_j(t)\Sigma_k(E_k(t))$ in Eq. 37 in a sample of countries over the period 1995–2000 by calculating the estimated damages from carbon emissions (now and in the future). They interpreted $\Sigma_j(g_j(t))$ to be the global shadow price of carbon in the atmosphere, and they arrived at a figure for $g_j(t)$ for each country j by attributing a share of the global damages to j. The estimated damages for country j were then subtracted from other investments in the calculation of comprehensive investment for j.

6.3 Capital Gains in Non-renewable Resources

Estimating wealth in open economies raises further problems. Oil exporting countries have been known to enjoy capital gains on their stocks underground. When the rental value of a non-renewable resource rises over time, owners of the resource stock enjoy capital-gains. However, as future consumers would expect to pay higher real prices, they experience a reduction in wealth, other things being equal. The impacts on wealth of a country's residents will depend on the extent to which the residents own (and sell) or consume (purchase) the resource in question. Either way, we need to estimate the resulting $\partial V/\partial t$ in Eq. 33.[40] It may be noted that in a closed economy there is no need to adjust wealth for capital gains or losses, since the future gains to owners will be offset by the losses to future consumers.

For each country, the capital gains on exhaustible resources equals the stock of the resource times the rate of increase of the shadow price (e.g., the rate of interest, if Hotelling's Rule prevails). Summing over all countries yields the total capital gains to that resource. The corresponding capital losses by purchasers must be equal to that sum. In principle, the latter should be allocated among individual countries in accordance with their future purchases of oil. In their empirical work, Arrow et al. (2007, 2008) estimated those losses by awarding each oil importing country a capital loss equal to the total capital losses to consumers times that country's share of current consumption of oil.

6.4 Population Change

Population is a capital asset. We have ignored that asset so far because population has been assumed to remain unchanged over time. Demographic change introduces complications to the analysis because we now have to add to the list of capital assets a set of (demographic) capital stocks; which means adding to the list of capital assets the size of each cohort in the population. For simplicity, let us assume that cohorts are identical in their preferences and abilities. Then the size of the population, $P(t)$, is the stock of the demographic asset. Arrow

[40] In three fine early studies, Asheim (1996), Sefton and Weale (1996), and Vincent et al (1997b) developed the concept of NNP in a trading economy possessing exhaustible resources.

et al. (2003b) developed the basics of the required analysis when a demographic theory is in hand. In the absence of a sound demographic theory, however, the analyst is forced to suppose that $P(t)$, like total factor productivity, changes exogenously over time. The effect of changes in P then appears in the term $\partial V/\partial t$ in Eq. 33. It remains to find a workable way to estimate that effect and isolate it from all other factors included in $\partial V/\partial t$. To do that it is expositionally simplest to assume that, excepting for population change, the resource allocation mechanism, α, is autonomous.

It could seem intuitive that, when population size changes, the criterion for sustainable development should be non-declining wealth *per capita*. It transpires that this is generally not true (Dasgupta 2001b; Arrow et al. 2003b). We now identify conditions under which that intuition is correct. We will discover that the conditions are very strong.

Write $c(t) = C(t)/P(t)$. The question arises as to how population should enter our conception of intergenerational well-being. In his classic work on optimum saving under constant population growth, Koopmans (1965, 1967) assumed the value function to be the present discounted sum of each generation's *average* felicity:

$$V(t) = \int_t^\infty \left[U(c(\tau))\mathrm{e}^{-\delta(\tau-t)}\right]\mathrm{d}\tau. \tag{39}$$

Meade (1955) had however already drawn attention to a deep problem with Expression 39: it discriminates against future people merely on the grounds that they will be members of generations of larger size.

An alternative (studied in the context of optimum saving, by Meade 1966; Mirrlees 1967; Arrow and Kurz 1970; Arrow et al. 2003b; and in the context of optimum saving and population, by Dasgupta 1969) is the present discounted sum of each generation's total felicity:

$$V(t) = \int_t^\infty \left[P(\tau)U(c(\tau))\mathrm{e}^{-\delta(\tau-t)}\right]\mathrm{d}\tau. \tag{40}$$

Arrow et al. (2003b) showed that if we wish to use the value function in Expression 40 for studying sustainable development, we would need to specify the level of consumption, c, at which $U(c) = 0$; implying that, when specifying U, we have only one degree of freedom (the scale of U). In the problem of optimum saving (as in Arrow and Kurz 1970) we wouldn't be required to do that, because we are free to choose both the scale and the level of U.[41]

It would be convenient in preparing national accounts if the level of U, not just its scale, could be freely chosen. So consider the following expression for intergenerational well-being:

$$V(t) = \int_t^\infty \left[P(\tau)U(c(\tau))\mathrm{e}^{-\delta(\tau-t)}\right]\mathrm{d}\tau \Big/ \int_t^\infty \left[P(\tau)\mathrm{e}^{-\delta(\tau-t)}\right]\mathrm{d}\tau. \tag{41}$$

The numerator in Expression 41 is Expression 40, whereas the denominator is the present discounted sum of each generation's population. Let us call the ethical theory on which Expression 41 is based, *dynamic average utilitarianism*.

Notice that the denominator in Expression 41 would play no role in policy evaluation *at* t (questions (C)–(D); Sect. 4.6), because the denominator would simply be a scale factor attached to Expression 40. But for sustainability analysis (question (B)) the denominator matters, because the evaluation there is undertaken *across* time.

[41] In the combined problem of optimum saving and population, Expression 40 does require of us to specify the value of c at which $U(c) = 0$. On this, see Meade (1955) and Dasgupta (1969).

Let $k(t) = K(t)/P(t)$, $l(t) = L(t)/P(t)$, and $u(t) = N(t)/P(t)$ be the *per capita* stocks of reproducible capital, human capital, and natural capital, respectively. Now write $\underline{s} = (k, l, u)$. From Expression 41, we have

$$V(t) = V(\underline{s}(t), P(t)). \tag{42}$$

Dasgupta (2001b) showed that if P grows (or declines) at a constant rate *and* if each of the equations that reflect the economy's accumulation process can be expressed in terms solely of *per capita* capital stocks, then $\partial V(t)/\partial P(t) = 0$. Under those conditions, we have

Proposition 13 *Development is sustained at t if and only if, when valued at constant shadow prices, (comprehensive) wealth per capita is non-decreasing at t.*

Proposition 13 has been used in applied studies on sustainable development (Dasgupta 2001b; Arrow et al. 2004, 2007, 2008; World Bank 2006). But the conditions under which the Proposition holds are empirically unacceptable. For example, the economy defined by Eqs. 3–6 violates the requirement that, when population is not constant the economy's accumulation process can be expressed in terms solely of *per capita* capital stocks. Unfortunately, Arrow et al. (2003b) have shown that the estimation of the shadow price of population will prove to be no easy matter. It would be useful to obtain a feel for the magnitude of the error we make when Proposition 12 is taken to be the theoretical basis for empirical work on sustainable development. To date we have no idea of the extent of that error.

6.5 Future Uncertainty

Predictably, future uncertainty creates additional problems for the study of sustainable development. Suppose, for simplicity, a Ramsey–Solow one-good economy. Population is assumed to remain constant. Technology is linear, but future productivity of capital is uncertain.

Intuition suggests that the uncertainty would be reflected in the shadow price of capital; so that, for example, if the elasticity of $U'(C)$ is greater than one, an increase in uncertainty would increase the shadow price of capital (Levhari and Srinivasan 1969). Marsili (2008) has shown that there are further complications for sustainability analysis.

Imagine that the productivity of capital follows a Wiener process with known parameters. Let E be the expectation operator. Given a resource allocation mechanism, α, we suppose that, when viewed from t, the stock of capital $K(t)$ follows the stochastic dynamics,

$$\mathrm{d}K = a(K, t)\mathrm{d}t + b(K, t)\mathrm{d}B, \tag{43}$$

where $K(t)$ is given, $a(K,t)$ is the expected value of (comprehensive) investment at t, and $\mathrm{d}B$ is the increment of the Wiener process (the risk term). In Eq. 43, $E[\mathrm{d}B] = 0$, $\mathrm{d}B^2 = E[\mathrm{d}B^2] = \mathrm{d}t$, and $a(K, t)$ and $b(K, t)$ are deterministic functions. (If the stochastic process is time-autonomous, a and b would not depend explicitly on t.)

Future consumption is determined by both α and the realizations of the Wiener process. Well-being is taken to be the expected value of $V(t)$ in Expression 7, which we write as $E[V(t)]$. We assume that $E[V(t)]$ exists.[42] Starting at t, consider the change in $E[V]$ in the small time interval $\mathrm{d}t$. K changes in that interval, by $\mathrm{d}K$, which is random. It follows from

[42] Marsili (2008) notes that if $U(C) \to -\infty$ as $C \to 0$ (as would be the case if the elasticity $U\prime(C)$ were constant and greater than one) and $b(K, t)$ is large, $E[V(t)]$ does not exist. For the corresponding result when time is discrete, see Dasgupta (2008).

Eq. 39 that

$$\begin{aligned}\mathrm{d}E\left[V(K(t),t)\right] &= E\left[V(K(t)+\mathrm{d}K,t)\right] - E\left[V(K(t),t)\right]\\ &\approx \{\partial E\left[V(K(t),t)\right]/\partial t\}\mathrm{d}t + \{\partial E\left[V(K(t),t)\right]/\partial K(t)\}E\left[\mathrm{d}K\right]\\ &\quad + \{\partial^2 E\left[V(K(t),t)\right]/\partial K^2(t)\}E\left[\mathrm{d}K^2\right]/2. \end{aligned} \tag{44}$$

But $E[\mathrm{d}K^2] \approx b^2(K(t),t)E[\mathrm{d}B^2] \approx b^2(K(t),t)\mathrm{d}t$. Taking the limit as $\mathrm{d}t \to 0$, Eq. 44 therefore reduces to

$$\begin{aligned}\mathrm{d}E\left[V(K(t),t)\right]/\mathrm{d}t &= \partial E\left[V(K(t),t)\right]/\partial t + a(K(t),t)\{\partial E\left[V(K(t),t)\right]/\partial K(t)\}\\ &\quad + b^2(K(t),t)\{\partial^2 E\left[V(K(t),t)\right]/\partial K^2(t)\}/2. \end{aligned} \tag{45}$$

The first two terms of the right hand side of Eq. 45 correspond to the right hand side of Eq. 33. The third term (what may be called the "Ito term") is perhaps not obvious, but it reflects the magnifying effect of risk over time. As $\partial^2 E[V(K(t),t)]/\partial K^2(t) < 0$ (this follows from the fact that $U''(C) < 0$), the effect of future uncertainty is to reduce the value of investment.

What are we to make of the third term, whose presence refutes the claim that changes in wealth are the right basis for judging sustainable development?[43]

The third term arises because the underlying process if hugely volatile (a point made explicit in the title of Marsili's paper). We know that if the Wiener process is the ultimate source of uncertainty, then the sample paths of K are continuous in t almost everywhere but differential nowhere in t. Marsili (2008) uses the Ito integral. In contrast, Weitzman (2003: Ch. 7) had argued in favour of the Stratonovich integral, which, if used, does not yield the "third term". If all this appears overly esoteric to social accountants, they should ask whether there is any particular evidence that the Wiener process is the correct one to use for modelling uncertainty in the factors that drive the macroeconomy. They could also ask whether continuous time is the right framework for creating green national accounts. If time is taken to be discrete, shadow prices would embody future uncertainty; and wealth, defined in terms of shadow prices, would continue to be the right basis for judging sustainable development. The subject of national accounts would seem to be one where the choice between continuous and discrete time matters.

7 Concluding Remarks

Lamenting that those working on the theory of capital and growth during the 1950s and 1960s had not addressed practical problems relating to world poverty but had instead engaged in debates on scholastic matters, Sen (1970) likened capital and growth theorists to someone poor who had collected money for food but had blown it all on alcohol instead. It has seemed to me for some time that there is more than a whiff of that same lack of concern with contemporary economic and ecological problems in the huge literature on the welfare economic theory of green national accounts. The underlying motivation there ought to have been the search for ways to judge the sustainability of development processes in the world we have begun to know. And yet, a large body of work has involved definitions, re-definitions, and finger exercises on the Hamiltonian in convex economies. In Sects. 4–5 I have shown that, in the simplest macro-economic model (constant population, time-autonomous resource allocation mechanism, no uncertainty), the appropriate criterion for assessing sustainable development

[43] I am most grateful to William ("Buzz") Brock for helpful correspondence on this.

is wealth, which is the social value of the stock of a comprehensive list of capital assets, where the value is measured in terms of shadow prices. That wealth is the correct basis with which to judge the sustainability of development paths follows directly from Lindahl's classic, but under-acknowledged, work. However, I have also argued that it is only a preliminary result, of little use in applied work.

So, in Sect. 6 I presented an account of what we are able to say when the very restrictive assumptions of the simplest macro-economic model are lifted. The findings collated there show that we know relatively little about the extensions that need to be made to the formulae for shadow prices if we are to apply the theory. The one powerful message the Millennium Ecosystem Assessment (Reid et al. 2005) has given to us economists is that estimating shadow prices of such vital assets as local and global ecosystems and the services they offer is now of central importance. The theory I have sketched shows that, in order to estimate shadow prices, particular attention will have to be paid to the resource allocation mechanisms that characterize our world and the fact that Nature's processes are highly non-linear. The estimates of sustainable development that we currently have (Dasgupta 2001b; Arrow et al. 2004, 2007, 2008; World Bank 2006) are far too crude for comfort. There is much left to be done.

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