Abstract:

We show that for every $\epsilon>0$ there is an absolute constant $c(\epsilon)>0$ such that the following is true:
The union of any n arithmetic progressions, each of length n, with pairwise distinct differences must consist of at least $c(\epsilon)n^{2-\epsilon}$ elements.
We show also that this type of bound is essentially best possible, as we observe n arithmetic progressions, each of length n, with pairwise distinct differences such that the cardinality of their union is $o(n^2)$.

We develop some number theoretical tools that are of independent interest.
In particular we give almost tight bounds on the following question: Given n distinct integers $a_{1},...,a_{n}$ at most how many pairs satisfy $a_{j}/a_{i} \in [n]$? More tight bounds on natural related problems will be presented.

This is a joint work with Shoni Gilboa.