Limit theorems for the Moebius function
Ya.G.Sinai
Princeton University

Moebius function (Mf) is one of the most important functions in number theory. The interest to its statistical properties grew up after the lectures by P.Sarnak several years ago. They follow from the structure of level sets of Mf. Cellarosi and myself, Baake and coauthors and other groups of people showed that the correlation functions of the square of Mf correspond to measures invariant under the shift with pure point spectrum. Most general results here were obtained recently by Cellarosi and Vinogradov. Correlation function of the Mf itself require more difficult analysis which is based on the use of signed measures. One of the corollaries is the proof of the Kac-Erdős type theorem for square-free numbers.

Shearing in time-changes of parabolic flows
Corinna Ulcigrai
University of Bristol

In this talk I will survey three results on parabolic flows which are based on a shearing mechanism, namely the proof of the Katok-Thouvenot conjecture on the nature of the spectrum for time-changes of classical horocycle flows (joint work with G. Forni), genericity of mixing for time-changes of Heisenberg nilflows (joint work with A. Avila and G. Forni) and mixing for smooth area-preserving flows on surfaces (Arnold's conjecture). We will highlight the similarities and differences of the shearing phenomena which are central in the proofs of these three results.
The horocycle flow and the Chacon transformation
Jean-Paul Thouvenot • Universite Pierre et Marie Curie

Kinetic transport in quasicrystals and Ratner's theorem
Jens Marklof • University of Bristol

Previous studies of kinetic transport in the Lorentz gas have been limited to cases where the scatterers are distributed at random (e.g. at the points of a spatial Poisson process or at the vertices of a Euclidean lattice. In this talk I will report on recent joint work with Andreas Strombergsson (Uppsala) on quasicrystalline scatterer configurations, which are non-periodic, yet strongly correlated. A famous example is the vertex set of the Penrose tiling. Our main result proves the existence of a limit distribution for the free path length, which answers a question of Wennberg. The limit distribution is characterised by a certain random variable on the space of higher dimensional lattices, and is distinctly different from the exponential distribution observed in the random setting. I will also discuss related results for other aperiodic scatterer configurations, such as the union of pairwise incommensurate lattices. The key ingredients in the proofs are equidistribution theorems on homogeneous spaces, which follow from Ratner's measure classification theorem.

Applications of Ratner’s rigidity theorem to number theory
Peter Sarnak • Princeton University
Local rigidity for partially hyperbolic and parabolic homogeneous actions

Anatole Katok • Penn State University

I will discuss the current state of local rigidity program for smooth actions of higher rank abelian groups. While for homogeneous hyperbolic actions local differentiable rigidity has been known since 1990ies and more recent work concerns global rigidity, for partially hyperbolic, and, somewhat surprisingly, for parabolic (unipotent) actions, local rigidity results appeared more recently.

In the partially hyperbolic case there are two very different approaches here: one based on harmonic analysis/theory of unitary group representation, and the other based on the study of the complicated web of stable and unstable foliations for various elements of the action. In the framework of the second approach some classical methods of algebraic K-theory turned out to be quite useful.

A joint work with Danijela Damjanovic provides a reduction of the local rigidity problem to the "linear" problem of classification of certain classes of cocycles over the unperturbed homogeneous action. The principal (semisimple) case concerns restrictions of the action of a maximal Cartan subgroup C of a semi-simple Lie group G on the homogenous space G/H where H is a cocompact lattice to a higher rank closed subgroup A of C. Previous work (Damjanovic--A.K., Damjanovic, Zhenqi Jenny Wang) dealt with various special cases and heavily used structural theory of simple Lie groups as well as highly non-trivial classical work in the algebraic K-theory. The method pursued in these papers was further developed by Kurt Vinhage who came close to completing the program (under the standard general position assumption on A); only some exceptional groups are left. However, Vinhage also made a remarkable discovery: the previous methods aimed at (and did) proofs of a stronger property than what was needed for cocycle rigidity. He found
an extremely elegant and fairly short general argument that completed the story and superseded previous work. Together with recent work of Z.J. Wang that provides a reduction of the twisted cases to the semisimple ones this essentially completes the program in the partially hyperbolic case.

In the parabolic case no direct geometric methods are available (in contrast with the fundamental work of Ratner on measure rigidity). Furthermore, in most cases one should only expect a parametric version of rigidity, as opposed to classification of all small perturbations of a homogeneous action. This is similar to the classical elliptic case (Arnold, Moser, Herman).

Up to now only one case has been fully treated (Damjanovic--A.K) namely the upper-triangular action on $\text{SL}(2,\mathbb{R}) \times \text{SL}(2,\mathbb{R})$. On the other hand, a general approach has been developed and various elements of it have been carried out in greater generality. Not surprisingly, one needs to use theory of unitary group representations in a serious way. Let me mention one result by Z.J. Wang that, although not definitive, is a remarkable discovery: the action of a maximal rank unipotent subgroup of $\text{SL}(n,\mathbb{R})$, $n>3$ satisfies infinitesimal version of absolute (not parametric!) local differentiable rigidity. This is completely new phenomenon that was not known before for any totally non-hyperbolic actions.

**Lattices in Adele groups**

**M. S. Raghunathan** • Indian Institute of Technology Bombay

In this talk we classify lattices in adele groups of algebraic groups. This is a straightforward extension of the arithmeticity results of Margulis. This classification has some connection with the Hasse Principle as well as the Platonov-Margulis conjecture to which we draw attention.
Soft counting
Marc Burger • ETH Zurich

The asymptotic of the number of integer points in euclidean balls on affine varieties homogeneous under the action of a semi-simple group has received considerable attention and has been established in a remarkable degree of generality by various people; it represents one of the spectacular applications of Ratner’s work. In this talk I want to take a different point of view, namely the one of determining the density at infinity of the integer points; one way of giving a precise meaning to this is suggested by the theory of Kleinian groups and Patterson’s construction of a density which is a tool to analyze their limit sets. We will apply these ideas to our setting and in particular exhibit in full generality certain restrictions on the possible supports of the densities defined on a suitable compactification of the affine variety. This is joint work with J. Portmann.

A correspondence principle for the primes
Tamar Ziegler • The Hebrew University

Continued fractions and values of binary quadratic forms
Shrikrishna Dani • Indian Institute of Technology Bombay

I will discuss some results on values of binary quadratic forms with real or complex coefficients over the set of integer pairs, respectively pairs of Gaussian integers, in terms of the continued fraction expansions of the slopes of the lines on which the forms vanish.
**Intrinsic Diophantine approximation on quadratic hypersurfaces and homogeneous flows**

Dmitry Kleinbock  
Brandeis University

The talk will be about a connection between the distribution of rational points on rational quadratic hypersurfaces and homogeneous dynamics. This work is joint with Lior Fishman, Keith Merrill and David Simmons.

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**The rate of Diophantine approximation by lattice orbits on homogeneous varieties**

Amos Nevo  
Technion

We will present upper and lower bounds on the speed of approximation in a wide range of natural Diophantine approximation problems. The upper and lower bounds coincide in many cases, giving rise to an optimal result. Our approach proceeds by establishing, more generally, bounds for the rate of distribution of dense orbits of a lattice subgroup in a connected Lie (or algebraic) group, acting on suitable homogeneous spaces. This approach is based on a quantitative duality principle for homogeneous spaces, and the upper bound on the rate of distribution we obtain is determined explicitly by the spectrum of the stability group in the automorphic representation. The rate is best possible when the representation in question is tempered, a condition which holds in a wide range of examples.

Based on Joint work with Anish Ghosh and Alex Gorodnik.
Symbolic dynamics for geodesic flows

Omri Sarig ▪ Weizmann Institute of Science

A famous 1969 paper of Marina Ratner implies that a geodesic flow on a compact surface with negative curvature can be coded as a suspension over a topological Markov shift. I will discuss a program for extending this result to other surfaces. (Joint with Y. Lima)

Badly approximable vectors on fractals

Barak Weiss ▪ Tel Aviv University

In joint work with David Simmons, we prove that certain natural fractal measures arising from iterated function systems of contracting similarities, give zero measure to badly approximable vectors. Previous results in this direction involved a reduction to the measure rigidity results of Lindenstrauss. Our approach involved a reduction to the analysis of stationary measures for certain semigroups acting on homogeneous spaces, extending work of Benoist and Quint.

The SL(2,R) action on moduli space

Alex Eskin ▪ University of Chicago

We prove some ergodic-theoretic rigidity properties of the action of SL(2; R) on the moduli space of compact Riemann surfaces. In particular, we show that any ergodic measure invariant under the action of the upper triangular subgroup of SL(2; R) is supported on an invariant affine submanifold. The main theorems are inspired by the results of several authors on unipotent flows on homogeneous spaces, and in particular by Ratner’s seminal work. This is joint work with Maryam Mirzakhani and Amir Mohammadi.
Ratner’s property, joinings and special flows over rotations

Mariusz Lemańczyk • Nicolaus Copernicus University

The talk will concentrate on so called Ratner’s property for flows. This property may be viewed as a particular way of divergence of orbits of close points; it was shown to hold for horocycle flows by M. Ratner in 1983 (under the name $H_p$-property). Basic joining consequences of Ratner’s property will be presented. Extensions of Ratner’s property, preserving joining consequences, will be discussed. Classes of special flows over irrational rotations enjoying (extended) Ratner’s property will be presented. Main part of the talk is based on my joint works with Krzysztof Fraczek.

Ratner’s work on rigidity of the horocycle flow and the low entropy method

Elon Lindenstrauss • The Hebrew University

In my talk I will discuss Ratner’s work on rigidity of the horocycle flow, particularly her classification of joinings of horocycle flows, and explain the relation between this work and the classification of measures invariant under diagonalizable actions.

Limit distributions of translates of algebraically defined measures

Nimish Shah • Ohio State University

Various counting and diophantine problems involving large groups of symmetries can be addressed by describing limiting distributions of stretching translates of certain algebraically defined measures on homo-
geneous spaces of Lie groups. Such limit measures turn out to be invariant under non-trivial unipotent subgroups, and hence they can analyzed using Ratner’s classification theorems and the linearization techniques. Finally one reduces the problem to linear dynamical questions involving finite dimensional representations of semisimple groups. We will describe different types of such problems and the corresponding linear dynamical results.

**Simplicity of the Lyapunov spectrum for some dynamical systems**

**Alex Furman** • University of Illinois at Chicago

Given an integrable matrix valued function $F: X \to SL_d(\mathbb{R})$ over an ergodic probability measure preserving system $(X, m, T)$, the asymptotic exponential expansion/contraction properties of the products $F_n(x) = F(T^{n-1}x) \cdots F(Tx)F(x)$ are characterized by a set of constants $\Lambda = (\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_d)$, called the Lyapunov spectrum. An important problem is to determine whether the spectrum is simple, i.e. the exponents are all distinct.

Simplicity of the spectrum for products of independent random variables was established by Guivarc’h-Raugi and Gol’dsheid-Margulis in 70s and 80s. More recently, simplicity of the spectrum for a certain system related to Teichmüller flow and Interval Exchange Transformations was conjectured by Kontsevich-Zorich and proved by Avila-Viana. Another case of simplicity of the spectrum appeared in the work of Eskin-Mirzakhani and Matheus.

In the talk I will describe a joint work with Uri Bader giving a "soft" proof of simplicity of the Lyapunov spectrum for a class of systems, including the above mentioned situations.
Kleinian groups and dynamics of unipotent flows

Hee Oh • Yale University

Joinings of semisimple higher rank actions

Manfred Einsledler • ETH Zurich

We describe the full classification of higher rank joinings for torus actions on homogeneous spaces defined by semisimple groups. The proof relies on entropy geometry, high and low entropy method. This is joint work with E. Lindenstrauss.

Diophantine approximation in nilpotent Lie groups

Emmanuel Breuillard • Université Paris-Sud 11

Pick k elements at random in a nilpotent Lie group and a word of length n with letters among those k elements. How close to the identity can this word be? The nilpotent Lie group is called Diophantine if this distance cannot be smaller than $n^\beta$ for some $\beta > 0$. We show that nilpotent Lie groups with step 1, 2, 3, 4 or 5 are Diophantine, but that there are counter-examples in step 6 and higher. Using the Kleinbock-Margulis method about diophantine approximation on manifolds we are also able to compute the optimal diophantine exponent $\beta$ and show that it is rational when the nilpotent Lie group is rational (joint work with M. Aka, N. De Saxce and L. Rosenzweig).
Quantitative multiple mixing
Alexander Gorodnik ▪ University of Bristol

We discuss asymptotic behavior of higher-order correlations for dynamical systems on homogeneous spaces. Using mixing properties of unipotent flows, we establish quantitative estimates which show that these correlations behave approximately like correlations of independent random variables. This is a joint work with M. Bjoerklund and M. Einsiedler.

Shapes of Lattices orthogonal to integer vectors
Uri Shapira ▪ Technion

Consider a euclidean sphere of large radius in the d-dimensional euclidean space and assume that the set L of integer points on this sphere is non-empty. What can be said about the finite collection of lattices obtained by intersecting the integer points in d-space with the hyperplanes orthogonal to the vectors in L? In a joint work with Aka and Einsiedler we show that these lattices - when put together in an appropriate space - equidistribute as the radius of the sphere increases (without constraint if \( d>5 \) and if \( d=3,4,5 \) with some congruence condition on the square of the radius).

Invariant measures on finite volume quotients
Yves Benoist ▪ Université Paris-Sud

Let G be a real semisimple Lie group with no compact factors, C a Zariski dense subgroup, and L a lattice in G. In a joint work with JF Quint, we prove that the set of ergodic C-invariant probability measures on G/L is both countable and compact.